**OREGON
DEPARTMENT
OF
TRANSPORTATION**

*GEOMETRONICS
200 Hawthorne Ave.,
B250
Salem, OR 97310
(503) 986-3103*

*Ron Singh, PLS
Chief of Surveys
(503) 986-3033*

*David Artman, PLS
Geometronics
(503) 986-3017*

*David W. Taylor, PLS
Geometronics
(503) 986-3034*

*Dave Brinton, PLS, WRE
Survey Operations
(503) 986-3035*
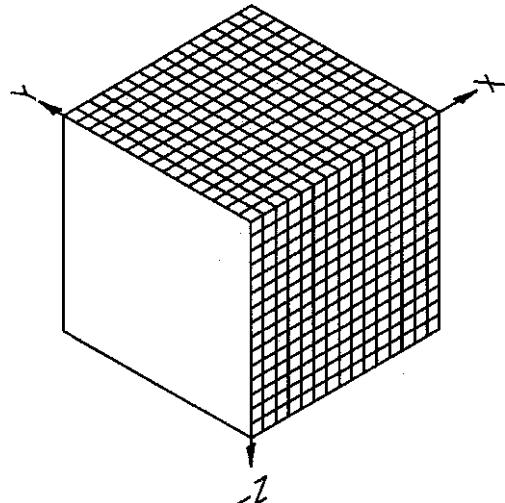
BASIC SURVEYING - THEORY AND PRACTICE

Ninth Annual Seminar
Presented by the
Oregon Department of Transportation
Geometronics Unit
February 15th - 17th, 2000
Bend, Oregon

Table of Contents

Types of Surveys

Surveying has to do with the determination of the relative spatial location of points on or near the surface of the earth. It is the art of measuring horizontal and vertical distances between objects, of measuring angles between lines, of determining the direction of lines, and of establishing points by predetermined angular and linear measurements. Along with the actual survey measurements are the mathematical calculations. Distances, angles, directions, locations, elevations, areas, and volumes are thus determined from the data of the survey. Survey data is portrayed graphically by the construction of maps, profiles, cross sections, and diagrams.

Types of Surveys:

Geodetic Surveying: The type of surveying that takes into account the true shape of the earth. These surveys are of high precision and extend over large areas.

Plane Surveying: The type of surveying in which the mean surface of the earth is considered as a plane, or in which its spheroidal shape is neglected, with regard to horizontal distances and directions.

Operations in Surveying:

Control Survey: Made to establish the horizontal and vertical positions of arbitrary points.

Boundary Survey: Made to determine the length and direction of land lines and to establish the position of these lines on the ground.

Topographic Survey: Made to gather data to produce a topographic map showing the configuration of the terrain and the location of natural and man-made objects.

Hydrographic Survey: The survey of bodies of water made for the purpose of navigation, water supply, or sub-aqueous construction.

Mining Survey: Made to control, locate and map underground and surface works related to mining operations.

Construction Survey: Made to lay out, locate and monitor public and private engineering works.

Route Survey: Refers to those control, topographic, and construction surveys necessary for the location and construction of highways, railroads, canals, transmission lines, and pipelines.

Photogrammetric Survey: Made to utilize the principles of aerial photo-grammetry, in which measurements made on photographs are used to determine the positions of photographed objects.

Review of Basic Trigonometry

<div style="text-align: right">**2**</div>

Pythagorean Theorem

Attributed to and named for the Greek philosopher and mathematician Pythagoras, the Pythagorean Theorem states:

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

(For the derivation of the Pythagorean Theorem, see the appendix.)
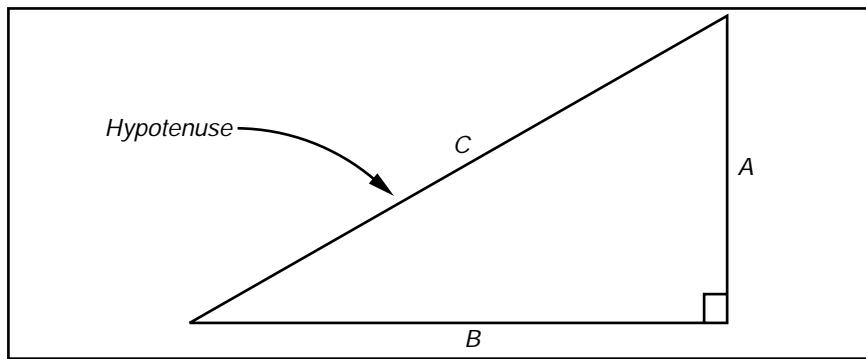
Figure 1

$$C^2 = A^2 + B^2$$

where: C is the hypotenuse (side opposite the right angle). A and B are the remaining sides.

Solving for C:

$$C^2 = A^2 + B^2$$

$$C = \sqrt{A^2 + B^2} \qquad \text{(take the square root of each side)}$$

Solving for A:

$$C^2 = A^2 + B^2$$

$$C^2 - B^2 = A^2 \qquad \text{(subtract } B^2 \text{ from each side)}$$

$$A^2 = C^2 - B^2 \qquad \text{(reverse the equation)}$$

$$A = \sqrt{C^2 - B^2} \qquad \text{(take the square root of each side)}$$

Solving for B:

$$B = \sqrt{C^2 - A^2} \qquad \text{(identical to solving for A)}$$

Using one of the forms of the Pythagorean Theorem on the previous page, solve for the unknown side in each triangle.

	Side A	Side B	Side C
1	3	4	
2		12	13
3	8		17
4		45	53
5	40	96	
6	36		111
7		5.6	6.5
8	3.5	8.4	
9	2.1		2.9

The first two are solved for you.

First Triangle

A = 3, B = 4, C = ?

Since C is the unknown, we solve for C.

$C^2 = A^2 + B^2$

$C = \sqrt{(3)^2 + (4)^2}$

$C = \sqrt{9 + 16}$

$C = \sqrt{25}$

$C = 5$

Second Triangle

A = ?, B = 12, C = 13

Since A is the unknown, we solve for A.

$A = \sqrt{C^2 - B^2}$

$A = \sqrt{(13)^2 - (12)^2}$

$A = \sqrt{169 - 144}$

$A = \sqrt{25}$

$A = 5$

Units of Angular Measurement

The most common angular units being employed in the United States is the Sexagesimal System.

This system uses angular notation in increments of 60 by dividing the circle into 360 degrees; degrees into 60 minutes; and minutes into 60 seconds. Each unit has a corresponding symbol: degrees are indicated by °; minutes by ´; and seconds by ˝.
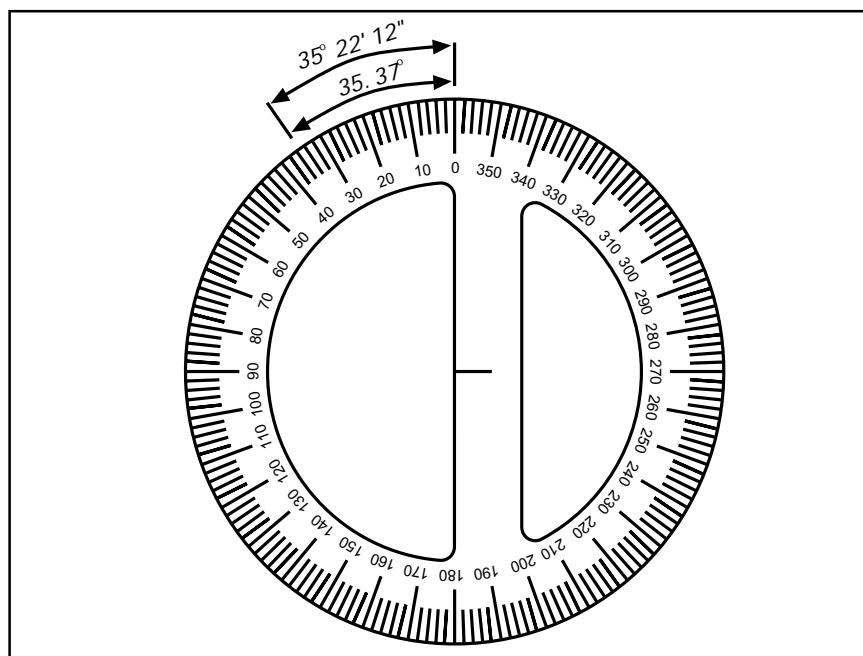
Figure 2

Therefore;

1 circle = 360° = 21,600´ = 1,296,000˝
1° = 60´ = 3600˝
1´ = 60˝

Usually angles are expressed in Degrees, Minutes, and Seconds as applicable, but can be expressed in any combination. For example, 35.37° 2122.2´ 127332˝, 34° 81´ 72˝, and 35° 22´ 12˝ all represent the same magnitude of angle. However, in the last form, which is the preferred notation, notice that minutes and seconds equal to or greater than 60 are carried over to the next larger unit and that degrees and minutes do not have decimals. Decimal seconds are acceptable.

For performing certain mathematical operations with angles, it is some-times easier to convert to decimal degrees first, perform the necessary math, then convert back to degrees, minutes, and seconds.

	Degrees - Minutes - Seconds	Decimal Degrees
10	23° 12′18″	
11		42.885°
12		63.545°
13	87° 58′48″	

23° + (12′ ÷ 60) + (18″ ÷ 3600) = 23.205°
42° + (0.885° x 60′) = 42° 53′ + (0.1′ x 60″) = 42° 53′ 06″

The primary unit of angular measurement in the metric system is the radian. A radian is defined as the angle between radius lines from either end of an arc of radius length.
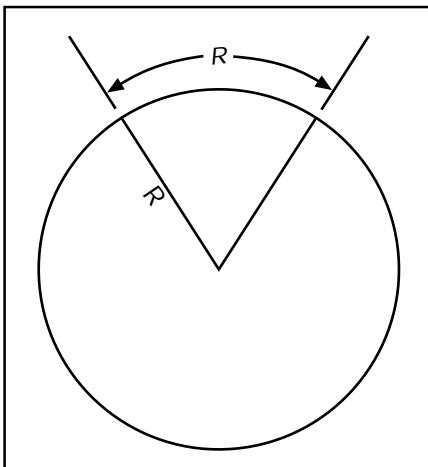
Figure 3

The circumference of a circle is twice the radius length times π, or C = 2πr. Therefore, 1 circle = 2π radians.

Since 1 circle = 360° = 2πrad.,
then 1 rad. = 360°/2π = 57.29578...°

The use of radians and the value of 57.29578° will be mentioned again when dealing with circular and spiral curves.

Another unit is the grad or gon. A grad is defined as 1/400 of a circle. The grad is widely used in much of the world as part of the metric system, even though the radian is the primary unit.

Ratios and Proportions

A ratio is a comparison of two values or quantities, and can be expressed in numerous ways. The ratio of 2 to 5, 2:5, 2 ÷ 5, or 2/5, are all expressions of the same ratio.

As a fraction, a ratio can be treated like any other fraction. The ratio is the quotient of the first value divided by the second value, and as such, can also be expressed as a decimal. In our example above, the ratio of 2 to 5 is 0.4000.

A proportion is a statement of equality between two ratios. Since the ratio of 2 to 5 is the same as the ratio of 4 to 10, we can say that the two ratios are a proportion. The same proportion can be expressed as 2:5 = 4:10, 2 ÷ 5 = 4 ÷ 10, or 2/5 = 4/10. Notice that 2/5 = 0.4000 = 4/10.

Find the value of x.

14) $\dfrac{1}{2}$ = $\dfrac{x}{4}$ $x =$

15) $\dfrac{2}{3}$ = $\dfrac{x}{12}$ $x =$

16) $\dfrac{5}{3}$ = $\dfrac{15}{x}$ $x =$

17) $\dfrac{789}{375}$ = x $x =$

18) 4.875 = $\dfrac{x}{124}$ $x =$

Definition of Trigonometric Functions

All trigonometric functions are simply ratios of one side of a right triangle to a second side of the same triangle, or one side over another side. The distinction between functions is which two sides are compared in the ratio.

The figure below illustrates the side opposite from and the side adjacent to Angle A, and the hypotenuse (the side opposite the right angle). The trigonometric functions of any angle are by definition:
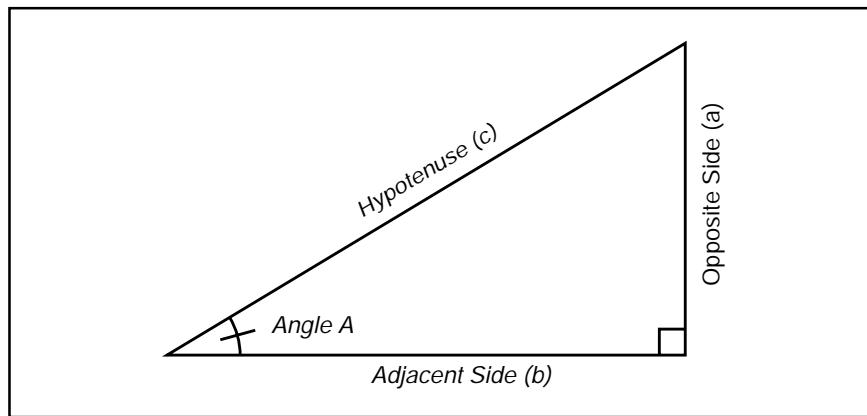
Figure 4

sine	=	Opposite Side / Hypotenuse
cosine	=	Adjacent Side / Hypotenuse
tangent	=	Opposite Side / Adjacent Side

and inverting each ratio, we have

cosecant	=	Hypotenuse / Opposite Side	=	1/sine
secant	=	Hypotenuse / Adjacent Side	=	1/cosine
cotangent	=	Adjacent Side / Opposite Side	=	1/tangent

Practice Problems:

19) Given a right triangle as shown above, and given side a = 3, side b = 4, and side c = 5, list the 6 trigonometric functions of angle A as a fraction and as a decimal.

20) Given side a = 42, side b = 56, and side c = 70, list the functions of angle A.

21) Given side a = 5, side b = 12, list the functions of angle A.

22) Given sin A = 0.2800, list cos A and tan A.

Trigonometric Functions of 45°

In the examples on the previous page, we developed trig functions for various angles A without determining the value of angle A. In order for trig functions to be of significant value, there must be a known correlation between the magnitude of the angle and the magnitude of the trigonometric functions.
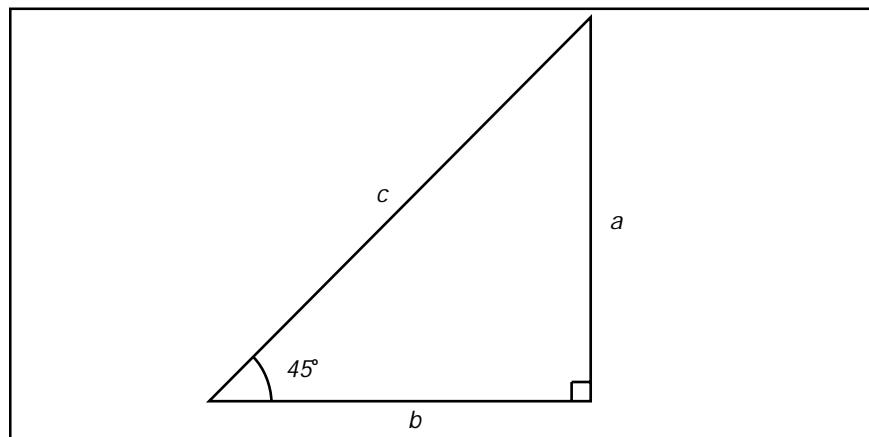
Figure 5

We can develop the functions for a 45° angle as follows:
Assume a 45° right triangle as shown. If we assign a value of 1 to side a, then we know that side b = 1 also.

Using the Pythagorean Theorem, side c = $\sqrt{2}$.

The sin, cos, and tan of 45° are:

$$\sin 45° = \frac{1}{\sqrt{2}} = 0.7071$$

$$\cos 45° = \frac{1}{\sqrt{2}} = 0.7071$$

$$\tan 45° = \frac{1}{1} = 1.0000$$

Trig Functions -vs- Size of the Triangle

On the previous page, we developed values for the trig functions of a 45° angle given assumed lengths of the sides of the triangle. But if the lengths of the sides were altered and the 45° angle held, would the trig functions remain unchanged? Let's find out.

Assuming a hypotenuse of c = 1, we can solve for the other two sides, again using the Pythagorean Theorem.

$$c = \sqrt{a^2 + b^2}$$

$$a = b$$

$$c = \sqrt{a^2 + a^2}$$

$$c^2 = 2a^2$$

$$a^2 = \frac{c^2}{2}$$

$$a = \sqrt{\frac{c^2}{2}}$$

$$a = \sqrt{\frac{1^2}{2}}$$

$$a = \sqrt{\frac{1}{2}} = 0.7071$$

$$b = a = 0.7071$$

Therefore our three primary trig functions are:

sin 45°	=	0.7071/1	=	0.7071
cos 45°	=	0.7071/1	=	0.7071
tan 45°	=	0.7071/0.7071	=	1.0000

We can see that the trig functions, (the ratios between the sides) are not dependent on the size of the triangle. Try developing the functions for the 45° angles in the following triangles.

23) hypotenuse (c) = 187,256

24) side opposite (a) = 0.0027

Trig Functions -vs- the Magnitude of the Angle

Now that we know that the size of a triangle does not affect the trig functions of its angles, let's find out what happens if we alter the shape of the triangle by increasing or decreasing the magnitude of the acute angles. The right angle cannot be modified since trig functions are the ratios of one side to another side of a **right** triangle.
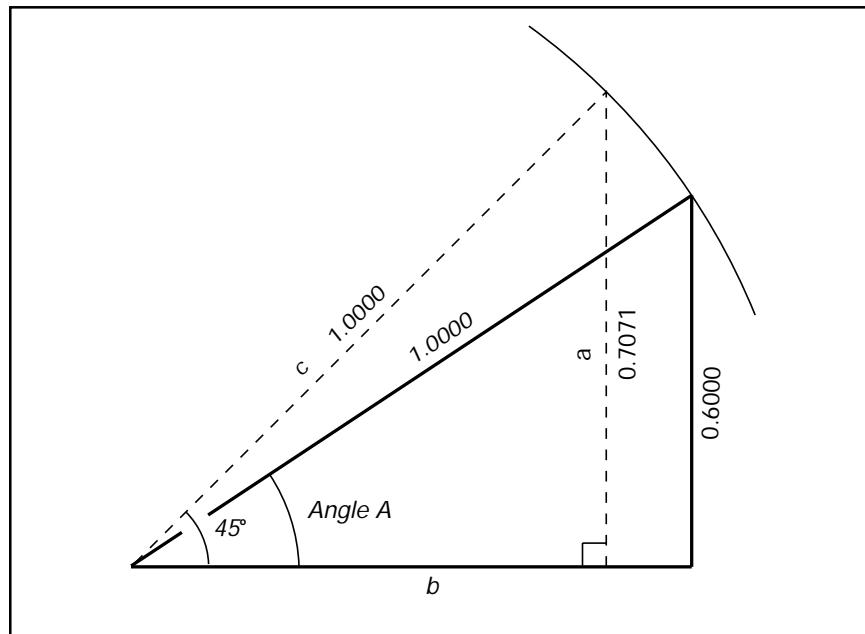
Figure 6

Let's start with our 45° triangle from the previous page, having a hypotenuse of 1.0000 and the other two sides of $\sqrt{1/2}$, or 0.7071 each. If we maintain the length of the hypotenuse while decreasing the angle A, the figure at the right shows that the side opposite also decreases, while the side adjacent to angle A increases.

Let's decrease angle A until side a is shortened from 0.7071 to 0.6000. At this point, we don't know the size of angle A, except that it is less than 45°. But knowing the lengths of sides a and c, we can determine side b to be 0.8000 and the trig functions of angle A to be:

$$
\begin{aligned}
\sin A &= 0.6000/1.0000 &= 0.6000 \neq \sin 45° &= 0.7071 \\
\cos A &= 0.8000/1.0000 &= 0.8000 \neq \cos 45° &= 0.7071 \\
\tan A &= 0.6000/0.8000 &= 0.7500 \neq \tan 45° &= 1.0000
\end{aligned}
$$

From this we know that changing the magnitude of the angle changes all of the trig functions associated with that angle.

Trigonometric Functions of 30° and 60°

We can develop trig functions for a 60° right triangle similar to the method used on the 45° triangle.
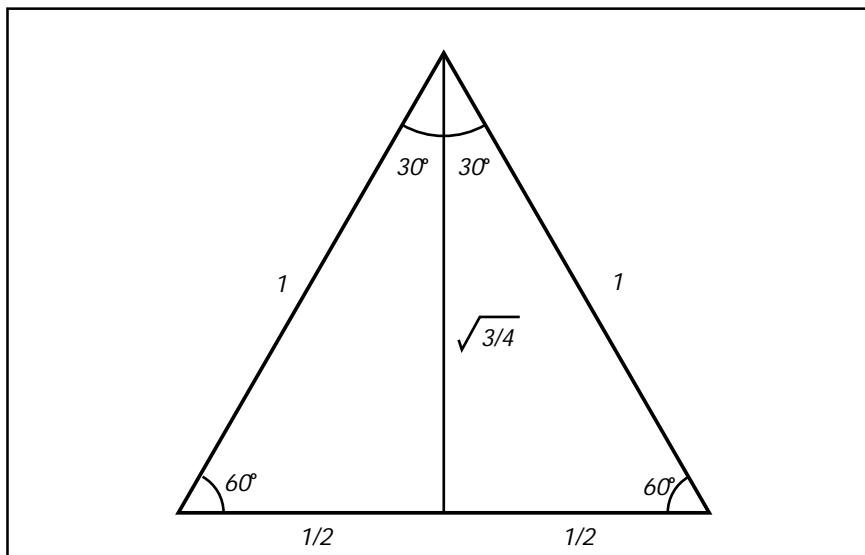
Figure 7

We can construct a 60° angle by creating an equilateral triangle having three 60° angles. If we assign a value of 1 to each side, bisect the triangle through the base and the vertex, we have a right, 60° triangle with a hypotenuse of 1 and the side adjacent to the 60° angle of 1/2. The side opposite then is $\sqrt{3/4}$ or 0.8660 and the trig functions are:

$$\sin 60° \; = \; \sqrt{\frac{3}{4}} \, /1 = \sqrt{3/4} \; = 0.8660$$

$$\cos 60° \; = \; \frac{1}{2} \, /1 = 1/2 = 0.5000$$

$$\tan 60° \; = \; \sqrt{\frac{3}{4}} \, / \, \frac{1}{2} \; = 2\sqrt{\frac{3}{4}} \; = 1.7321$$

25) Having bisected the 60° angle at the vertex of our equilateral triangle, we now have two 30° angles. Based on the above figure, what are the trig functions of 30°?

$\sin 30° \; =$

$\cos 30° \; =$

$\tan 30° \; =$

Cofunctions

Of the six trigonometric functions that have been discussed, three have the prefix "co" in their names. The "co" functions of sine, tangent, and secant are cosine, cotangent, and cosecant respectively.

Any function of an acute angle is equal to the cofunction of its complementary angle. Complementary angles are two angles whose sum is 90°.

Since the two acute angles in any right triangle are complementary, the functions of one angle are equal to the cofunctions of the other. We found this in our work with 30° and 60° angles.

sine 30°	=	0.5000	=	cosine 60°
cosine 30°	=	0.8660	=	sine 60°
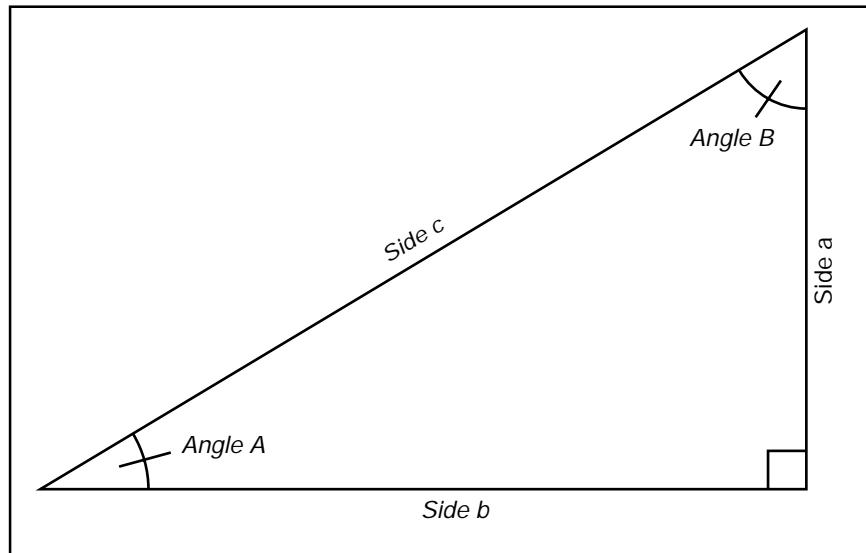
Figure 8

Sin A	=	Cos B	=	Side a / Side c
Cos A	=	Sin B	=	Side b / Side c
Tan A	=	Cot B	=	Side a / Side b
Cot A	=	Tan B	=	Side b / Side a
Sec A	=	Csc B	=	Side c / Side b
Csc A	=	Sec B	=	Side c / Side a

Trigonometric Functions for Angles Greater than 90°

So far we have only dealt with functions for angles between 0° and 90°. Angles outside of this range cannot be included in a right triangle as specified in the earlier definitions of the functions.
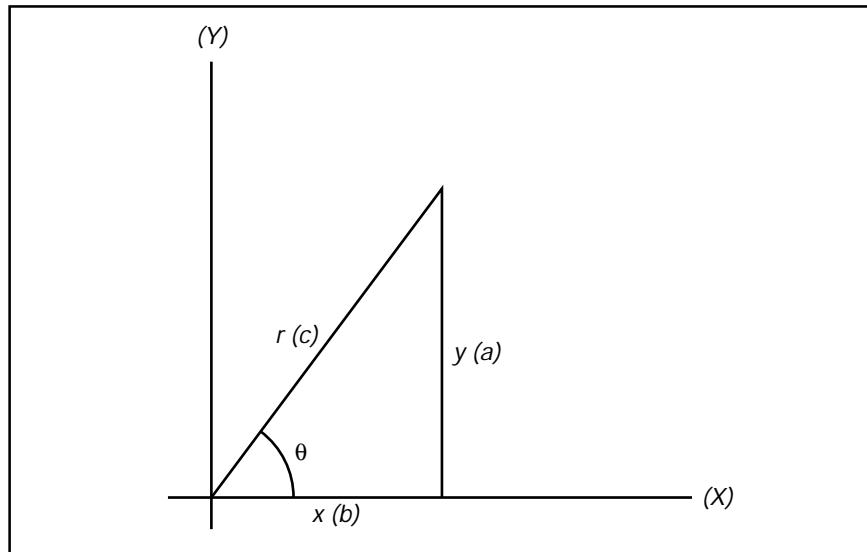
Figure 9

However, if we place the right triangle on x,y axes as shown, we can rewrite the trigonometric functions. The hypotenuse becomes r, or the radial distance from the origin. The adjacent side becomes x, or the distance along the x-axis. The opposite side becomes y, or the right angle distance from the x-axis.

The trigonometric functions of θ then are:

$$\sin\theta = \text{opposite side / hypotenuse} \quad = \text{a/c} \; = \; \text{y/r}$$
$$\cos\theta = \text{adjacent side / hypotenuse} \quad = \text{b/c} \; = \; \text{x/r}$$
$$\tan\theta = \text{opposite side / adjacent side} \quad = \text{a/b} \; = \; \text{y/x}$$

With these definitions, we can increase or decrease θ by any amount we choose and still have x and y, either or both of which may be positive, negative, or zero. The radial distance, r, is always considered positive in the θ direction.

Since x and/or y may be negative, the trigonometric functions may also be negative based on the algebraic signs of x and y.

Algebraic Signs of
the Trigonometric Functions in each Quadrant

Using the definitions on the previous page, we can determine the values of the functions for each angle shown below. List the Sine, Cosine, and Tangent of each angle in both fractional and decimal form. Three are already done.

26) Sinθ = 3/5 = 0.6000

27) Cosθ =

28) Tanθ =

29) Sin 180 - θ =

30) Cos 180 - θ = -4/5 = -0.800

31) Tan 180 - θ =

32) Sin 180 + θ =

33) Cos 180 + θ =

34) Tan 180 + θ = -3/-4 = 0.7500

35) Sin 360 - θ =

36) Cos 360 - θ =

37) Tan 360 - θ =

Figure 10

Notice that the angle θ becomes a "reference angle" for each of the other three, and that the magnitude of the functions are the same for each angle with only the algebraic signs differing.

38) The signs of the functions in quadrant 1 are all positive. Show the signs of the others in the chart below.

	Quad 1	Quad 2	Quad 3	Quad 4
Sin	+			
Cos	+			
Tan	+			

Trigonometric Functions of Quadrantal Angles

The Quadrantal Angles (0°, 90°, 180°, 270°, and 360°) have unique functions in that all cases, the opposite side (x) and the adjacent side (y) always meet the following condition. One is equal to plus or minus the hypotenuse (r) and the other is equal to zero. This combination can only yield three possible values for the trig functions: 0, ±1, and an unidentified value (division by 0).

39) List the values of x and y as 0 or r and show the resulting functions below.

	X=	Y=	Sine	Cosine	Tangent
0°					
90°					
180°					
270°					
360°					

A closer look at the undefined values is in order. The tangent of 90° has an x value of 0 causing a division by zero. If we consider the tangent of an angle slightly less than 90°, we have a y value very near to r and a very small x value, both positive. Dividing by a very small number yields a large function. The closer the angle gets to 90°, the smaller the x value becomes, the closer the y value becomes to r, and the larger the tangent function becomes. When the angle reaches 90°, the tangent approaches infinity, or tan 90°= ∞. But if an angle slightly larger than 90° is evaluated in a similar way, division by a very small negative x occurs, creating a tangent function approaching negative infinity, or tan 90°= -∞. In reality, the function is undefined and we express it as tan 90°= ±∞.
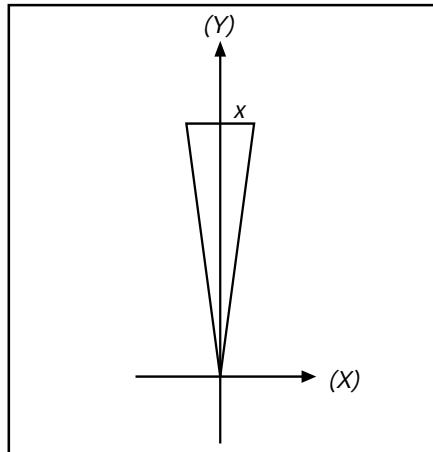
Figure 11

Values of Trigonometric Functions

There are several possible ways to determine the values of the trigonometric functions of a given angle:

1) The infinite series for Sine and Cosine functions.

2) Interpolation of values from published Trigonometric Tables.

3) Select the appropriate button on a scientific calculator.

The first is long and involved and beyond the scope of this course. Both (1) and (2) have become obsolete due to (3). We will assume that our little electronic wonders will return the proper value when a function is calculated. Notice that only three functions exist on most electronic calculators, as the others can be expressed as reciprocals of those shown, or otherwise easily reached.

While each angle has only one value for each of its trigonometric functions, exercise problems 26-37 reveal that more than one angle can have the same trigonometric values. $\text{Sin}\theta$ and $\text{Sin } 180° - \theta$, $\text{Cos}\theta$ and $\text{Cos } 360° - \theta$, and $\text{Tan}\theta$ and $\text{Tan } 180° + \theta$ are just three examples. Your calculator cannot determine which value is truly correct when taking an inverse or *arc* function (determining an angle from a function) so it will return a value between -90° and +180° depending on the function. It will be up to the individual to evaluate whether that is the correct value for the particular situation.

Practice Problems:

40) Determine the missing side of a 30° right triangle with a hypotenuse of 6.

41) Determine the angles in a 3,4,5 triangle.

42) Measured slope distance is 86.95 feet at +8.5°. What is the horizontal distance and the elevation difference?

Distance Measuring (Chaining)

<div style="text-align:right">**3**</div>

History of Chaining

The Egyptians were one of the first known people to use some form of chaining in both land surveying and construction surveying. On a wall in the tomb of Thebes and carved on a stone coffin are drawings of rope stretchers measuring a field of grain. The Great Pyramid of Gizeh (2900 B.C.) was constructed with an error of 8 inches in it's 750 foot base. This is an error of 1 in 1000 on each side.

English mathematician Edmund Gunter (1581-1626) gave to the world not only the words *cosine* and *cotangent,* and the discovery of *magnetic variation,* but the measuring device called the Gunter's chain shown below. Edmund also gave us the *acre* which is 10 square chains.

Figure 12

The Gunter's chain is 1/80th of a mile or 66 feet long. It is composed of 100 links, with a link being 0.66 feet or 7.92 inches long. Each link is a steel rod bent into a tight loop on each end and connected to the next link with a small steel ring.

Starting in the early 1900's surveyors started using steel tapes to measure distances. These devices are still called "chains" to this day. The terms "chaining" and "chainman" are also legacies from the era of the Gunter's chain.

Procedures for Chaining

It must be remembered in surveying, that under most circumstances, all distances are presumed to be horizontal distances and not surface distances. This dictates that every field measurement taken be either measured horizontally or, if not, reduced to a horizontal distance mathematically.

In many instances, it is easiest to simply measure the horizontal distance by keeping both ends of the chain at the same elevation. This is not difficult if there is less than five feet or so of elevation change between points. A hand level or "pea gun" is very helpful for maintaining the horizontal position of the chain when "level chaining." A pointed weight on the end of a string called a "plumb bob" is used to carry the location of the point on the ground up to the elevated chain by simply suspending the plumb bob from the chain such that the point of the plumb bob hangs directly above the point on the ground.

When the difference in elevation along the measurement becomes too great for level chaining, other methods are called for. One option, "break chaining", involves simply breaking the measurement into two or more measurements that can be chained level. This works well for measurements along a gentle slope where a reasonable distance can be measured between break chaining points.

In some cases, it becomes impractical to break chain. When the slope becomes so steep that frequent chaining points are required, a vertical surface must be measured across, or intermediate chaining points are not readily accessible, it may be more desirable to determine the horizontal distance indirectly. The most frequently used method is "slope chaining", where the distance along the slope is measured, the slope rate is determined, and the horizontal distance calculated.

Slope rate angles are measured as Vertical Angles (+/- from horizontal), or as Zenith Angles (Measured downward from a imaginary point on the celestial sphere directly above the instrument).

When the vertical angle (v) is used, the horizontal distance (HD) is calculated by multiplying the slope distance (SD) by the cosine of the vertical angle. This is the solution of a right triangle for the side adjacent (horizontal distance) with the hypotenuse (slope distance) known.

From basic trigonometry, we know that;

   cosine = Adjacent Side ∕ Hypotenuse

Substituting the known values, we have;

   $\cos(v) = HD ∕ SD$

Solving for HD by multiplying both sides of the equation by SD, we get;

   $HD = SD\cos(v)$

If the zenith angle (z) is measured rather than the vertical angle, the calculations are nearly identical. The only variation is that the zenith angle is the complimentary angle of the vertical angle, so the sine function must be used. The formula is;

   $HD = SD\sin(z)$

Errors in Chaining

Every measurement made with a mechanical device is subject to any error that could possibly be caused by the condition of the device or by the procedure used in taking the measurement. Some of the more common sources of errors are the standards to which the chain was manufactured, any damage to the chain, sag in the chain, variation in the tension on the chain, and changes in the temperature of the chain. With proper care of the chain and reasonable effort made with each use, the effects of these errors can be kept within acceptable tolerances for all but the most precise measurements. When necessary, however, each of these conditions can be compensated for mathematically if they are monitored and compared to a known standard.

Proportional Errors

When a chain is manufactured, it is intended to be a specific length, plus or minus some tolerance. It may or may not actually meet those specifications. When a field measurement is taken, the acceptable error may be more or less than what the chain was designed for. For high precision work, we need to measure several known distances and determine if this chain is the proper length. If not, we need next to determine if the error is in one or more specific locations along the chain or if the error is proportional along the length. If a known 50 foot distance is measured to be 49.995 feet and a known 100 foot distance to be 99.99 feet, all measurements made with that chain should be multiplied by a factor of 100 / 99.99 (known distance over measured distance).

Constant Errors

If a chain has been kinked or broken and spliced back together, there is a good chance that there will be a consistent error for any distances measured using that portion of the chain. This error needs to be added or subtracted as appropriate each time.

Sag Correction

When a chain is suspended from each end and not supported along it's length, the weight of the chain causes it to sag and pull the two ends toward each other. It is impossible to exert enough outward force to fully overcome the sag. For all measurements, adequate tension should be applied to minimize the effective shortening of the chain. For precise measurements, a correction should be applied using the formula given in the appendix.

Tension Correction

While a certain amount of tension is desirable to help offset the sag effect, it will also stretch the chain. Steel is generally thought of as not being very easily stretched and indeed it is not. That is one of the reasons it is used for making chains. But steel will still stretch to some degree if tension is applied. When a chain is checked against a known distance, the applied tension should be controlled. Subsequent precise measurements should be made using the same tension, or if not, a correction should be applied. The formula for this is also in the appendix.

Temperature Correction

Whatever material is used to make a chain, that material will expand and contract with any change in temperature. Some materials are more affected than others, but every chain will change length somewhat if warmed or cooled. If precise measurements are needed, an adjustment needs to be made for the change in temperature between the current temperature and the temperature at the time the chain was checked against a known distance. This formula is also in the appendix.

Distance Measuring
(Electronic Distance Meters)

In the early 1950's the first Electronic Distance Measuring (EDM) equipment were developed. These primarily consisted of electro-optical (light waves) and electromagnetic (microwave) instruments. They were bulky, heavy and expensive. The typical EDM today uses the electro-optical principle. They are small, reasonably light weight, highly accurate, but still expensive. This chapter will focus on electro-optical instruments only.
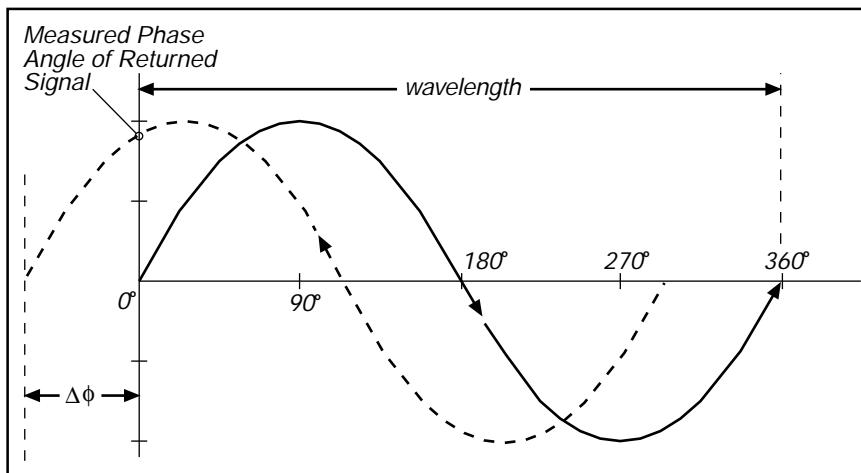
Figure 13

Basic Principle

To measure any distance, you simply compare it to a *known* or *calibrated* distance; for example by using a scale or tape to measure the length of an object. In EDM's the same comparison principle is used. The calibrated distance, in this case, is the wavelength of the modulation on a carrier wave.

Modern EDM's use the precision of a Quartz Crystal Oscillator and the measurement of *phase-shift* to determine the distance.

The EDM is set up at one end of the distance to be measured and a reflector at the other end. The EDM generates an infrared continuous-wave carrier beam, which is modulated by an *electronic shutter* (Quartz crystal oscillator). This beam is then transmitted through the aiming optics to the reflector. The reflector returns the beam to the receiving optics, where the incoming light is converted to an electrical signal, allowing a phase comparison between transmitted and received signals. The amount by which the transmitted and received wavelengths are out of phase, can be measured electronically and registered on a meter to within a millimeter or two.

Suppose the distance to be measured is an exact multiple (*n*) of 10 m. The optical path however, will be 2*n* x 10 m (this is the double distance). As the total path is 2*n* x 10 m, the total phase delay is *n* x 360°. Each 10 m wavelength represents a full 360° phase delay. The phase difference between a transmitted and received beam is also *n* x 360°, which cannot be distinguished from a 0° phase shift.

In general, the distance to be measured may be expressed as *n* x 10 + *d*. The total phase delay Φ between transmitted and received signals becomes:

$$\Phi = n \text{ x } 360° + \Delta\Phi$$

In which ΔΦ equals the phase delay due to the distance *d*. As *n* x 360° is equivalent to 0° for a phase meter, the angle can be measured and will represent d according to the relation.

$$d = \frac{\Delta\Phi}{360°} \text{ x } 10 \text{ m}$$

Most EDM's use four modulation frequencies to measure long distances without ambiguity.

The EDM first transmits a particular frequency generating a 10 m wavelength which determines the fraction of 10 m measured, it then switches to a 100 m wavelength to determine the fraction of 100 m, 1000 m wavelength to determine the fraction of 1000 m and 10 000 m wavelength to determine the fraction of 10 000 m.

Wavelengh Generated	Resolve Distance
10 m	xxx3.210
100m	xx73.210
1000 m	x273.210
10 000	1273.210

The final distance is then converted and displayed in the units desired.

In the latest generation instruments, this process is done in less that 2 seconds.

Accuracy Specifications

EDM specifications are usually given as a standard deviation. The specification given is a two part quantity. A constant uncertainty (regardless of the distance measured) and a parts-per-million (ppm) term (proportionate to the distance measured).

EDM Make and Model	Constant	Proportion
Wild D14 and D14L	+/– 5 mm	+/– 5 ppm
Wild D15 and D15S	+/– 3 mm	+/– 2 ppm
Wild/Leica DI1600	+/– 3 mm	+/– 2 ppm
Wild DI2000	+/– 1 mm	+/– 1 ppm
TCA 1800	+/– 1 mm	+/– 2 ppm

For short distances, the constant part of the error is significant and could exceed the normal errors of ordinary taping. As in angle measurements, short traverse sides should be avoided.

Environmental Correction

The procedures used to measure distances depends entirely on an accurate modulation wavelength. When the infrared beam is transmitted through the air, it is affected by the atmospheric conditions that exist. To correct for these conditions, the barometric pressure and temperature must be measured and the appropriate corrections made. All EDM's come with charts and formulas to compute this *PPM* correction. This value can usually be stored in the instrument. Optionally, the PPM correction could by applied later instead. An example ppm chart is shown in the appendix.

Prisms

The reflector, or prism, is a corner cube of glass in which the sides are perpendicular to a very close tolerance. It has the characteristic that incident light is reflected parallel to itself, thus returning the beam to the source. This is called a retrodirective prism or *retro reflector.*

These reflectors have a so-called "effective center". The location of the center is not geometrically obvious because light travels slower through glass than air. The effective center will be behind the prism itself and is generally not over the station occupied. Thus there is a *reflector constant* or *prism constant* to be subtracted from the measurement. Some manufacturers shift the center of the EDM forward the same amount as the prism offset to yield a zero constant. All Wild/Leica EDM's are shifted forward by 35 mm.
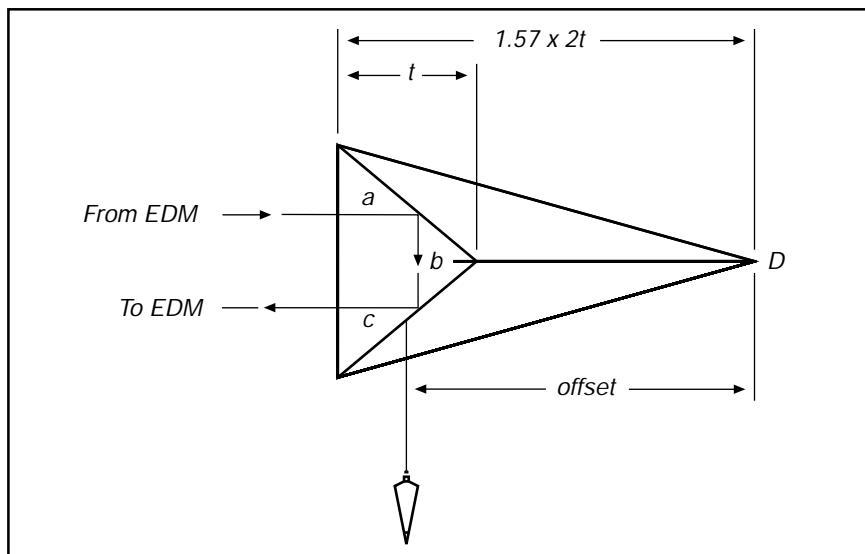
Figure 14

Always use prisms designed for your EDM system. Virtually all electro-optical EDM's of today mount to the theodolite and, depending on the type of the mount (scope, yoke, built in, etc), the prism housing has to be designed accordingly.

Slope Reduction

EDM instruments all measure (line of sight) slope distances only. In surveying, we are primarily concerned with horizontal distances. Therefore, this slope distance must be reduced to a horizontal distance. Most EDM's have the ability to make these computations through the use of a keyboard or by passing the raw distance to an electronic theodolite which in turn performs the function. For short distances, a simple *right triangle* reduction may be applied:

$$\text{Horizontal Distance} = s \times \sin(z)$$

When larger distances are involved, the earth's curvature and atmospheric refraction need to be taken into account. The equations are as follows:

Horizontal Distance in meters = $s(\sin z - E_1 \cos z)$

Vertical Distance in meters = $s(\cos z + E \sin z)$

Where:

$$E_1 = \frac{0.929 \; s(\sin z)}{6\;372\;000 \; m}$$

$$E = \frac{0.429 \; s(\sin z)}{6\;372\;000 \; m}$$

$$s = \text{slope distance in meters}$$

$$z = \text{zenith angle}$$

The horizontal distance equation uses the instrument elevation as the datum. If the sight is long and steeply inclined, a reciprocal observation (from the other end) would give a different answer. Each can be reduced to sea level by multiplying them by the following factor:

$$\frac{6\;372\;000}{6\;372\;000 + H}$$

$$H = \text{station elevation in meters}$$

Where *H* is the station elevation in meters. A more modern approach producing better results is the use of *reciprocal zenith* computations where the zenith angles and slope distances are measured from both ends of the line. The difference in elevation is the average of the elevations and the correction for earth curvature and refraction cancels.

Angle Measuring

Measuring distances alone in surveying does not establish the location of an object. We need to locate the object in 3 dimensions. To accomplish that we need:

1. Horizontal length (distance)

2. Difference in height (elevation)

3. Angular direction.

This chapter discusses the measurement of both horizontal and vertical angles.

An *angle* is defined as the difference in direction between two convergent lines. A *horizontal angle* is formed by the directions to two objects in a horizontal plane. A *vertical angle* is formed by two intersecting lines in a vertical plane, one of these lines horizontal. A *zenith angle* is the complementary angle to the vertical angle and is formed by two intersecting lines in a vertical plane, one of these lines directed toward the zenith.
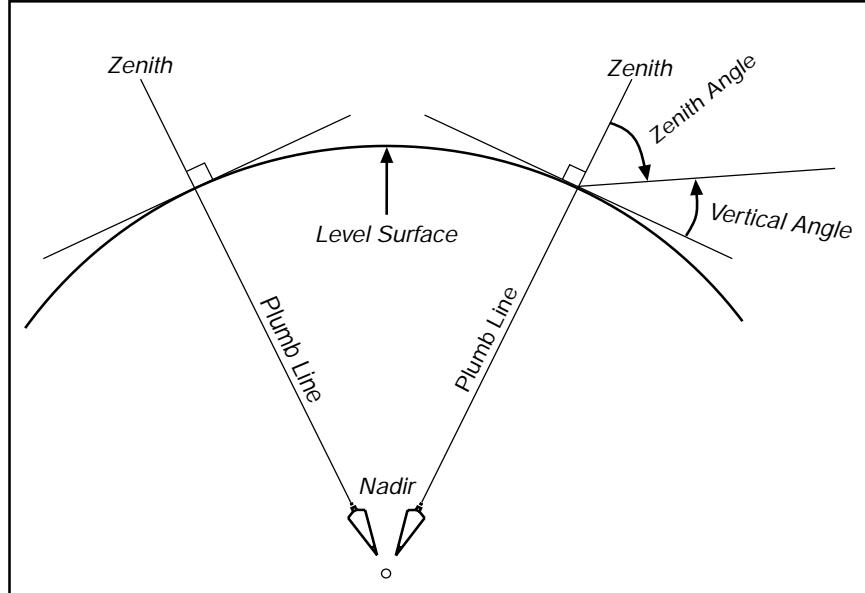
Figure 15

Types of Measured Angles

Interior angles are measured clockwise or counter-clockwise between two adjacent lines on the inside of a closed polygon figure.

Exterior angles are measured clockwise or counter-clockwise between two adjacent lines on the outside of a closed polygon figure.

Deflection angles, right or left, are measured from an extension of the preceding course and the ahead line. It must be noted when the deflection is right (R) or left (L).

Angles to the right are turned from the *back* line in a clockwise or right hand direction to the ahead line. **This is ODOT's standard.**

Angles to the left are turned from the back line in a counter-clockwise or left hand direction to the ahead line.

Angles are normally measured with a transit or a theodolite, but a compass may be used for reconnaissance work.
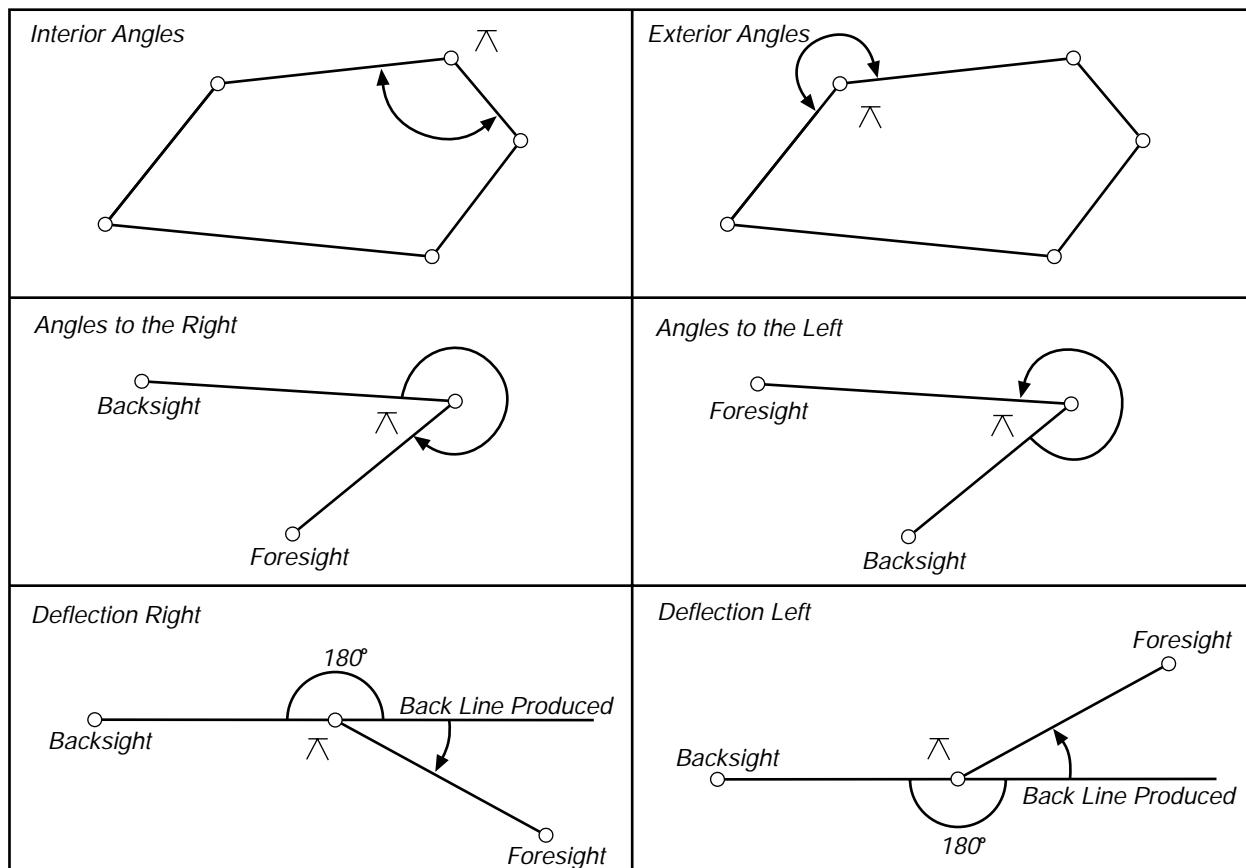
Figure 16

A *Transit* is a surveying instrument having a horizontal circle divided into degrees, minutes, and seconds. It has a vertical circle or arc. Transits are used to measure horizontal and vertical angles. The graduated circles (plates) are on the outside of the instrument and angles have to be read by using a vernier.
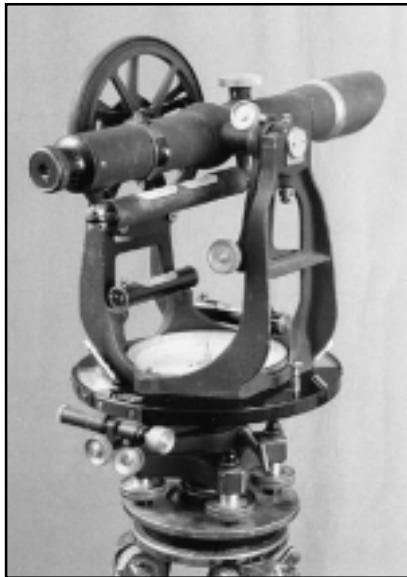
Figure 18

A *Theodolite* is a precision surveying instrument; consisting of an alidade with a telescope and an accurately graduated circle; and equipped with the necessary levels and optical-reading circles. The glass horizontal and vertical circles, optical-reading system, and all mechanical parts are enclosed in an alidade section along with 3 leveling screws contained in a detachable base or *tribrach.*

Figure 17

As surveyors we must know several relationships between an angular value and its corresponding subtended distance.

Surveyors must strive to maintain a balance in precision for angular and linear measurements. If distances in a survey are to be measured with a relative precision of 1 part in 20,000, the angular error should be limited to 10 seconds or smaller.

Comparison of Angular and Linear Errors

Standard error of angular measurement	Linear error in 1000 Units. (Feet – Meters)	Accuracy Ratio
05'	1.454	1:688
01'	0.291	1:3436
30"	0.145	1:6897
20"	0.097	1:10,309
10"	0.048	1:20,833
05"	0.024	1:41,667
01"	0.005	1:200,000

Repeating Instruments

All transits and some theodolites are constructed with a double vertical axis. They are equipped with upper and lower circle clamps and tangent screws, sometimes referred to as upper and lower motions. The lower clamp secures the horizontal circle to the base of the instrument while the upper clamp secures the circle to the alidade (the upper body of the instrument).Through the use of both clamps and their accompanying tangent (fine adjustment) screws, these instruments can be used to measure angles by repetition.

In ODOT's survey equipment fleet, the *Wild T16's are Repeating Instruments.* These instruments have a modified design providing tangent screw and one traditional type clamp that actually secures the alidade to the base. This clamp acts as either upper or lower motion depending on the position of the locking lever located near the tangent screw. With the lever in the down position, the circle is clamped to the alidade and the lock and tangent screw function as a lower motion. When the lever is moved to the up position, the circle is released from the alidade and allowed to rest on the base of the instrument, causing the clamp and tangent to function as an upper motion.

Measuring Angles with a Repeating Instrument

Repeated measurements of an angle increase accuracy over that obtained from a single measurement. A horizontal angle may be accumulated and the sum can be read with the same precision as the single value. When this sum is divided by the number of repetitions, the resulting angle has a precision that exceeds the nominal least count of the instrument.

To measure an *angle* set by repetition:

1. Set zero on the horizontal plate, and lock the upper motion.

2. Release the lower motion, sight the backsight, lock the lower motion, and perfect the sighting with the lower tangent screw.

3. Release the upper motion, turn to the foresight, lock the upper motion, and perfect the sighting.

4. Record the horizontal angle.

5. Release the lower motion, plunge (invert) the scope and point to the backsight in the reverse position, lock the lower motion, and perfect the sighting.

6. Release the upper motion, turn to the foresight, lock the upper motion, and perfect the sighting.

7. Record the double angle. Compute the mean angle.

8. If further accuracy is desired continue this process until 6 angles are accumulated. Divide the result by 6 and compare the result to the mean of the first 2. If they agree within 6 seconds accept the angle. Otherwise redo the set. **In ODOT, we are required to turn (6) angles for Traverse and (2) for other less critical control points.**

The expected accuracy of a measurement, as computed, is in direct proportion to the number of observations. However, factors limiting accuracy include, eccentricity in instrument centers, errors in the plate graduations, instrument wear, setting up and pointing the instrument, and reading the scale or vernier. A practical limit to the number of repetitions is about 6 or 8, beyond which there is little or no appreciable increase in accuracy.

Example of an angle set turned by a REPEATING instrument:

STA	ANGLE RIGHT	
FS		
	1) 39° 21. 6'	
	2) 78° 43. 3'	
⊼	m) 39° 21' 39"	
	6) 236° 09. 9'	
	M) 39° 21' 39"	
BS		
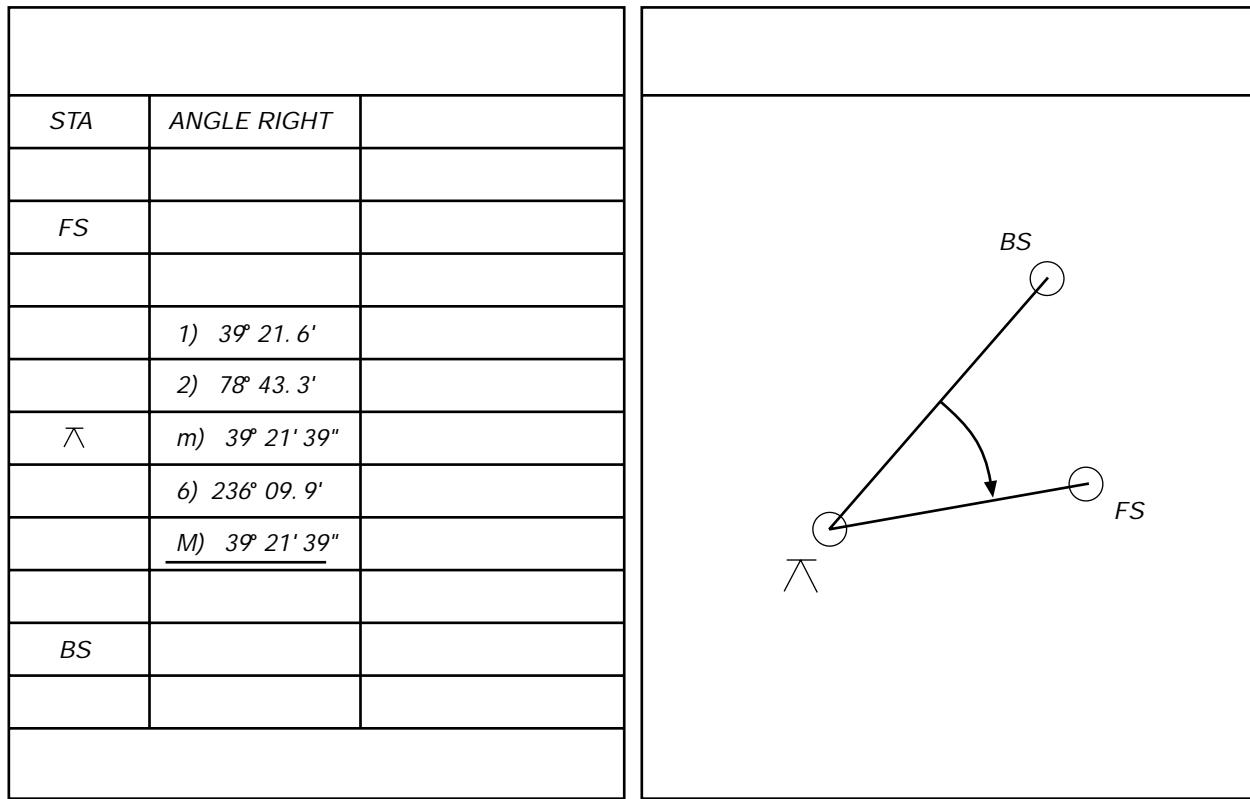
Figure 19

Measuring Angles with a Directional Instrument

A *Directional Theodolite* is not equipped with a lower motion. It is constructed with a single vertical axis and cannot accumulate angles. It does, however, have a horizontal circle positioning drive to coarsely orient the horizontal circle in any desired position. In ODOT's survey equipment fleet, *the T2000's, T1600's and T1610's are all Directional Instruments.*

A directional theodolite is more precise than a repeating theodolite. Directions, rather than angles are read. After sighting on a point, the line direction from the instrument to the target is noted. When a pointing is taken on the next mark, the difference in directions between them is the included angle.

To measure an *angle set* with a directional theodolite:

1. Point to the backsight in the *direct* position, lock on the target and record the plate reading. *Although not mathematically necessary, we set the horizontal circle to zero to simplify the calculations and to aid in any necessary debugging of the data.*

2. Loosen the horizontal motion and turn to the foresight. Lock the horizontal motion, perfect the sighting, then record the horizontal plate reading.

3. Loosen both horizontal and vertical motions, plunge the scope and point to the foresight. Again (in the reverse position) lock the horizontal motion, perfect the sighting and record the horizontal plate reading.

4. Loosen the horizontal motion and turn to the backsight, lock the horizontal motion, perfect the sighting and record the horizontal plate reading.

This completes one set. Depending on the accuracy required additional sets should be turned.

In ODOT, we are required to turn (2) sets for Traverse angles and (1) set for other less critical control points.

To reduce the notes, mean the direct and reverse observations to the backsight and foresight. Compute the mean direction to the foresight by subtracting the value of the meaned initial direction (backsight) to get final directions. If any set varies from the mean of all sets by more than +/- 5 seconds, reject that set and re-observe that particular set.

Example of two angle sets turned by a Directional Instrument:

STA	PLATE READING	ADJ. PLATE READING	SET ANGLE	FINAL ANGLE
1 - (DIR) BS	0° 00" 00"	0° 00" 02"		
4 - (REV) BS	180° 00' 04"		38° 21' 39.5"	
2 - (DIR) FS	38° 21' 43"	38° 21' 41.5"		
3 - (REV) FS	218° 21' 40"			
				_38° 21' 40"_
1 - (DIR) BS	0° 00' 00"	-00° 00' 01.5"		
4 - (REV) BS	179° 59' 57"		38° 21' 40.5"	
2 - (DIR) FS	38° 21' 42"	38° 21' 39"		
3 - (REV) FS	218° 21' 36"			
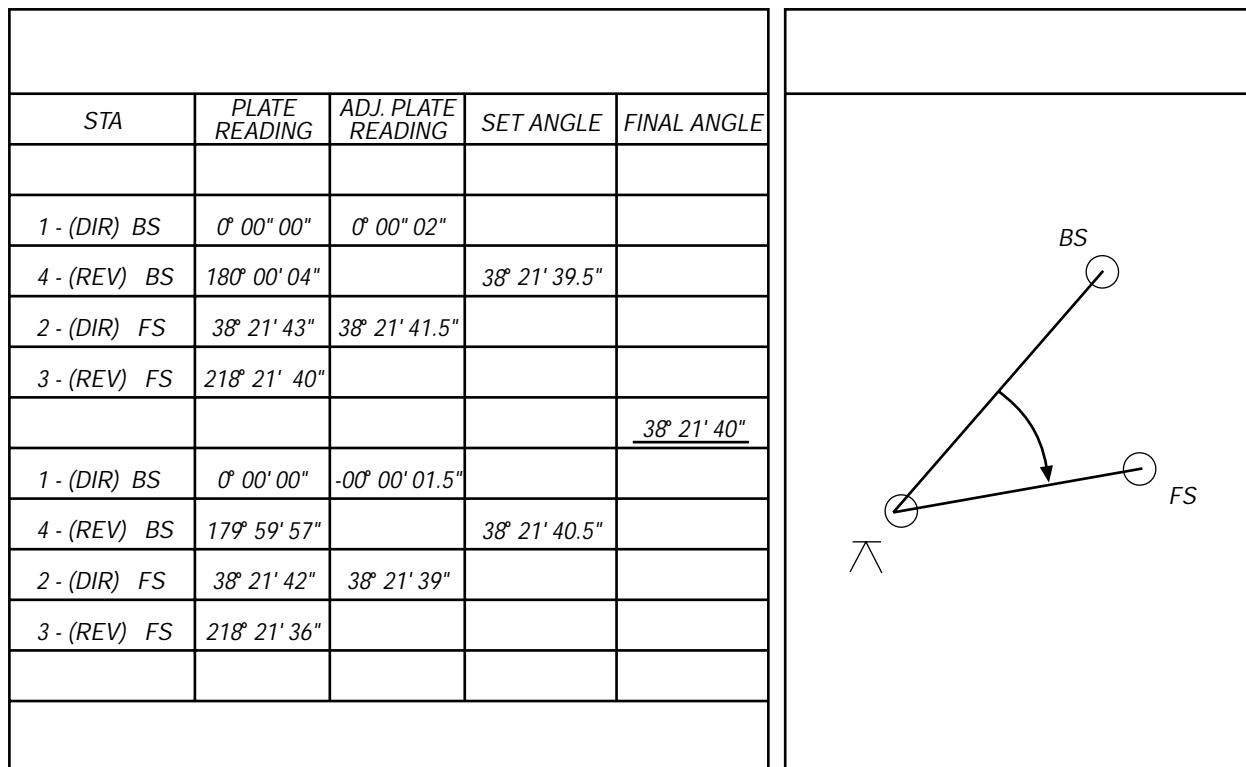
Figure 20

Zenith Angles

Unlike transits, theodolites are not equipped with a telescope level. Modern theodolites have an indexing system that utilizes an *automatic compensator* responding to the influence of gravity.

When the theodolite is properly leveled, the compensator is free to bring the vertical circle index to its true position. Automatic compensators are generally of two types:

1.   *Mechanical,* whereby a suspended pendulum controls prisms directing light rays of the optical-reading system.

2.   *Optical,* in which the optical path is reflected from the level surface of a liquid.

To measure a zenith angle:

1.   Point the instrument to the target object in a direct position.

2.   Lock the vertical motion, perfect the sighting and record the zenith angle.

3.   Loosen both the horizontal and vertical motions, plunge the scope, rotate the alidade 180° and re-point to the target in the reverse position.

4.   Lock the vertical motion, perfect the pointing and record the zenith angle.

A mean zenith angle is obtained by first adding the direct and reverse readings to obtain the algebraic difference between their sum and 360°; then dividing this difference by 2, and algebraically adding the result to the first (direct) series measurement.

The result is the zenith angle corrected for any residual index error.

Example:

Direct	83° 28'16"
Reverse	276° 31'38"
Sum	359° 59'54"
360° minus Sum	00° 00'06"
Half Value (error)	00° 00'03"
Plus Original Angle	83° 28'16"
FINAL ANGLE	83° 28'19"

Exercise - **Calculate Final Angles from given**

STA	ANGLE RIGHT	
FS		
	1) 57° 02.2'	
	2) 114° 04.5'	
⊼		
	6) 342° 13.7'	
BS		
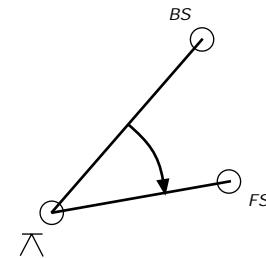
STA	PLATE READING	ADJ. PLATE READING	SET ANGLE	FINAL ANGLE
1 - (DIR) BS	0° 00" 00"			
4 - (REV) BS	179° 59' 54"			
2 - (DIR) FS	127° 57' 14"			
3 - (REV) FS	307° 57' 21"			
1 - (DIR) BS	359° 59' 58"			
4 - (REV) BS	180° 00' 07"			
2 - (DIR) FS	127° 57' 11"			
3 - (REV) FS	307° 57' 16"			
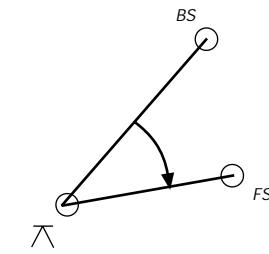
Direct Zenith Angle	102° 12 '45 "
Reverse Zenith Angle	257° 47 '21 "
Final Zenith Angle	

Bearings and Azimuths

The *Relative* directions of lines connecting survey points may be obtained in a variety of ways. The figure below on the left shows lines intersecting at a point. The direction of any line with respect to an adjacent line is given by the horizontal angle between the 2 lines and the direction of rotation.

The figure on the right shows the same system of lines but with all the angles measured from a *line of reference* (O-M). The direction of any line with respect to the line of reference is given by the angle between the lines and its direction of rotation.

The line of reference we use is a Meridian

There are several types of meridians: Astronomical or True, Magnetic, Grid, and Assumed.

Figure on left: *Directions by angles* — lines D, A, B, C intersecting at O with angles a1, a2, a3, a4.

Figure on right: *Directions referred to meridian* — meridian line O-M with lines D, A, B, C and angles b1, b2, b3, b4.
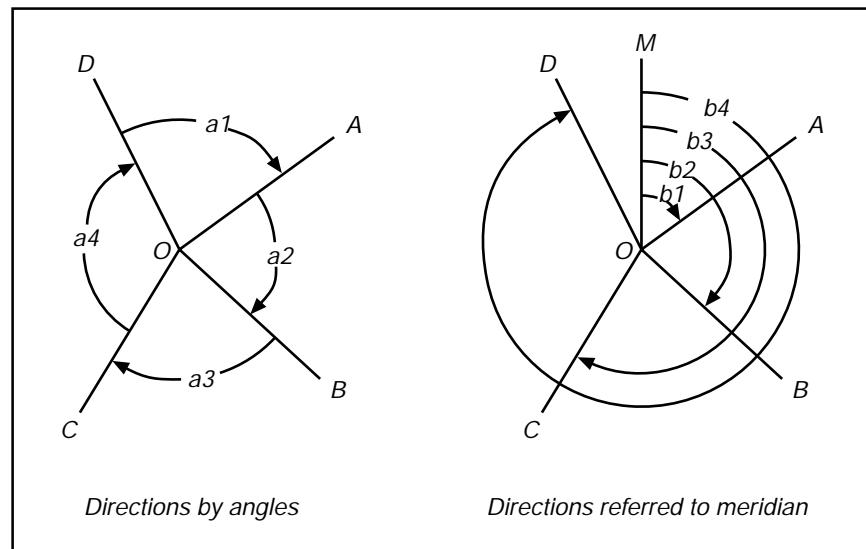
Figure 21

Astronomical or True Meridians

A plane passing through a point on the surface of the earth and containing the earth's axis of rotation defines the astronomical or true meridian at that point. Astronomical meridians are determined by observing the position of the sun or a star. For a given point on the earth, its direction is always the same and therefore directions referred to the astronomical or true meridian remain unchanged. This makes it a good line of reference.
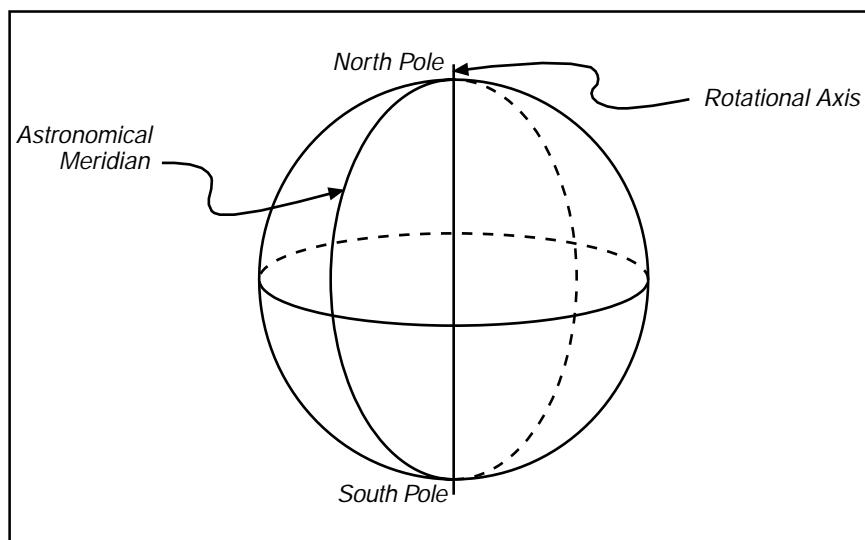
Figure 22

Convergence

Astronomical or true meridians on the surface of the earth are lines of geographic longitude and they converge toward each other at the poles. The amount of convergence between meridians depends on the distance from the equator and the longitude between the meridians.

Magnetic Meridian

A magnetic meridian lies parallel with the magnetic lines of force of the earth. The earth acts very much like a bar magnet with a north magnetic pole located considerably south of the north pole defined by the earth's rotational axis. The magnetic pole is not fixed in position, but rather changes its position continually. The direction of a magnetized needle defines the magnetic meridian at that point at that time. Because the magnetic meridian changes as magnetic north changes, magnetic meridians do not make good lines of reference.
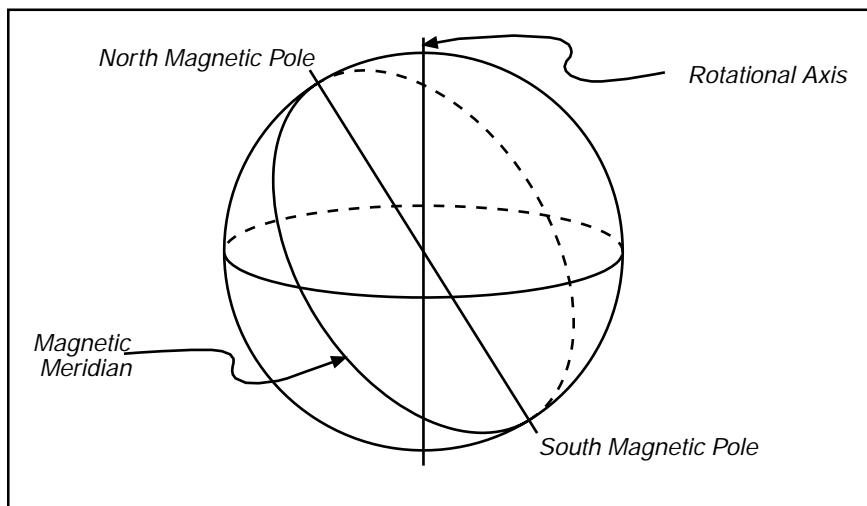
Figure 23

Grid Meridians

In plane surveys it is convenient to perform the work in a rectangular *XY* coordinate system in which one central meridian coincides with a true meridian. All remaining meridians are parallel to this central true meridian. This eliminates the need to calculate the convergence of meridians when determining positions of points in the system. The methods of plane surveying, assume that all measurements are projected to a horizontal plane and that all meridians are parallel straight lines. These are known as grid meridians.

The Oregon Coordinate System is a grid system.
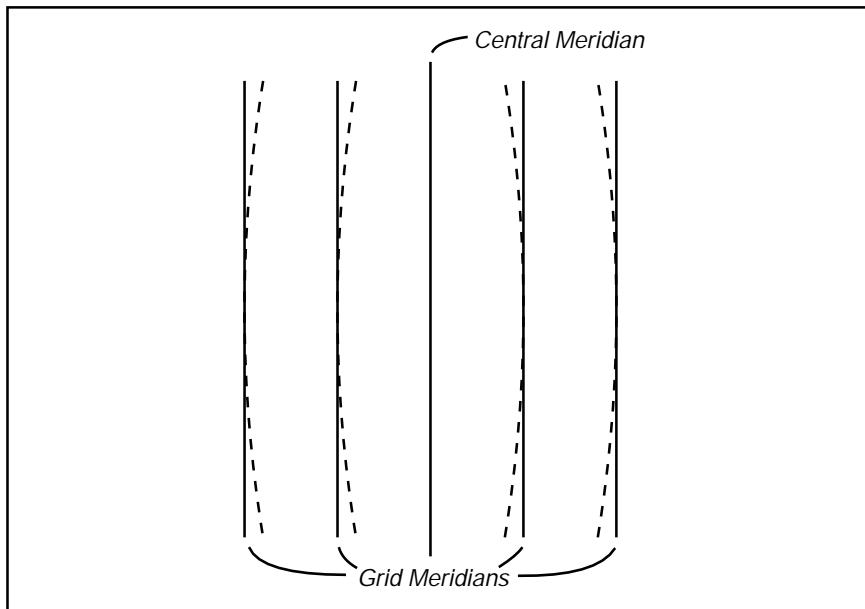
Figure 24

Assumed Meridians

On certain types of localized surveying, it may not be necessary to establish a true, magnetic, or grid direction. However it is usually desirable to have some basis for establishing relative directions within the current survey. This may be done by establishing an assumed meridian.

An assumed meridian is an arbitrary direction assigned to some line in the survey from which all other lines are referenced. This could be a line between two property monuments, the centerline of a tangent piece of roadway, or even the line between two points set for that purpose.

The important point to remember about assumed meridians is that they have no relationship to any other meridian and thus the survey cannot be readily (if at all) related to other surveys. Also, if the original monuments are disturbed, the direction may not be reproducible.

It is good practice when assuming a direction to avoid directions that might appear to be true. If assuming a direction on a line that runs generally north and south, do not assume a north direction, as some future surveyor may mistakenly use your direction as true.

Azimuths

The azimuth of a line on the ground is its horizontal angle measured from the *meridian* to the line. Azimuth gives the direction of the line with respect to the meridian. It is usually measured in a clockwise direction with respect to either the north meridian or the south meridian. In plane surveying, azimuths are generally measured from the north.

When using azimuths, one needs to designate whether the azimuth is from the north or the south.

Azimuths are called true (astronomical) azimuths, magnetic azimuths, grid azimuths, or assumed azimuths depending on the type of meridian referenced.

Azimuths may have values between 0 and 360 degrees.

The azimuth from the North for each line is:

Line	Azimuth
O – A	54°
O – B	133°
O – C	211°
O – D	334°
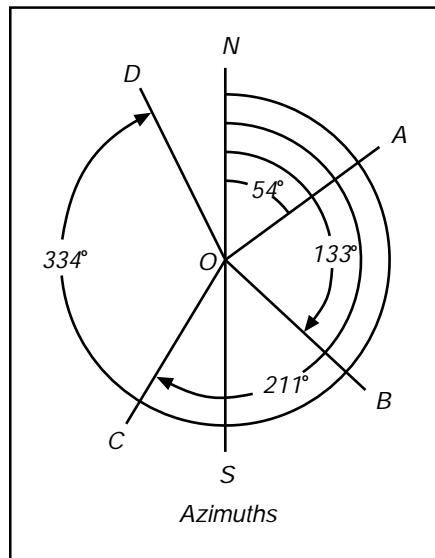
Azimuths

Figure 25

Using the deflection angles shown, calculate North azimuths of the lines.
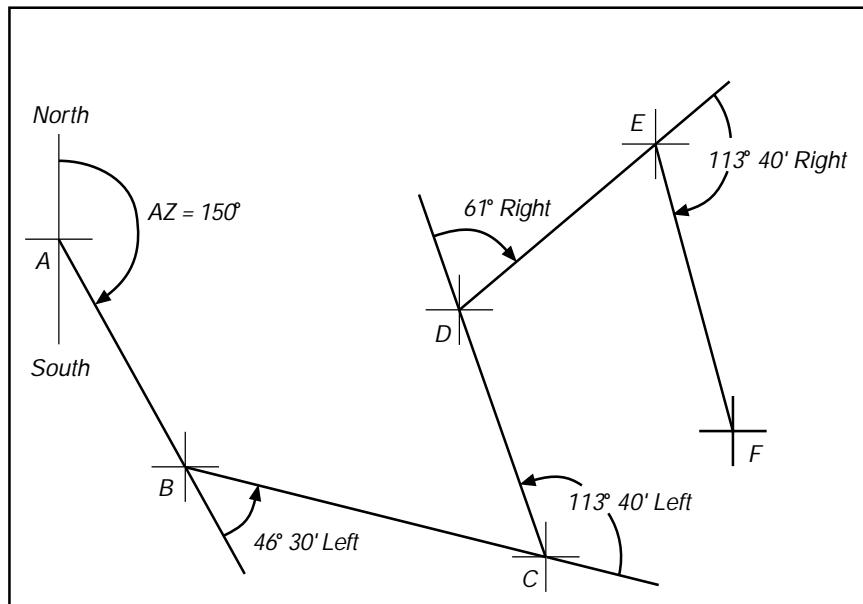
Figure 26

Line	Azimuth
A – B	
B – C	
C – D	
D – E	
E – F	

Bearings

The bearing of a line also gives the direction of a line with respect to the reference meridian. The bearing states whether the angle is measured from the north or the south and also whether the angle is measured toward the east or west. For example, if a line has a bearing of S 47° E, the bearing angle 47° is measured from the south meridian eastward.

A stated bearing is said to be a true bearing, a magnetic bearing, an assumed bearing, or a grid bearing, according to the type of meridian referenced.
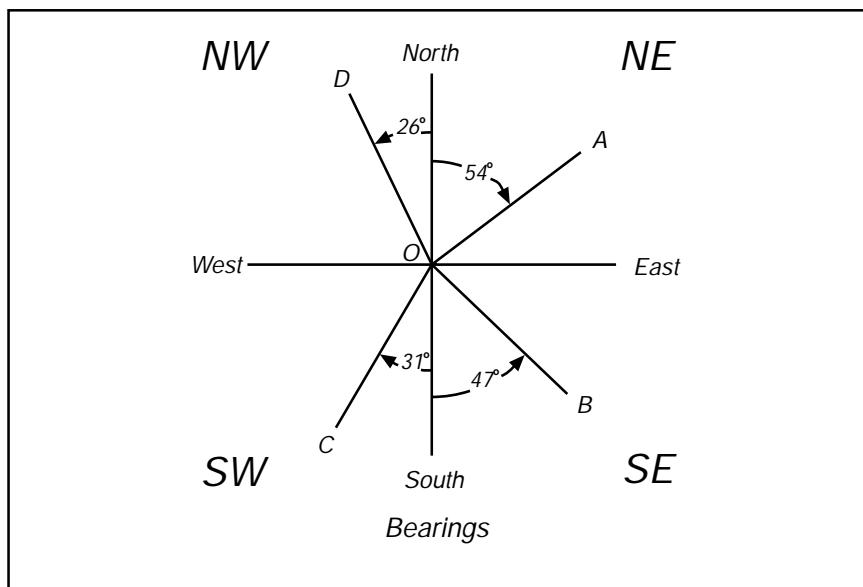
Figure 27

Line	Bearing
O – A	N 54° E
O – B	S 47° E
O – C	S 31° W
O – D	N 26° W

For the figure below, calculate the bearings for each line.
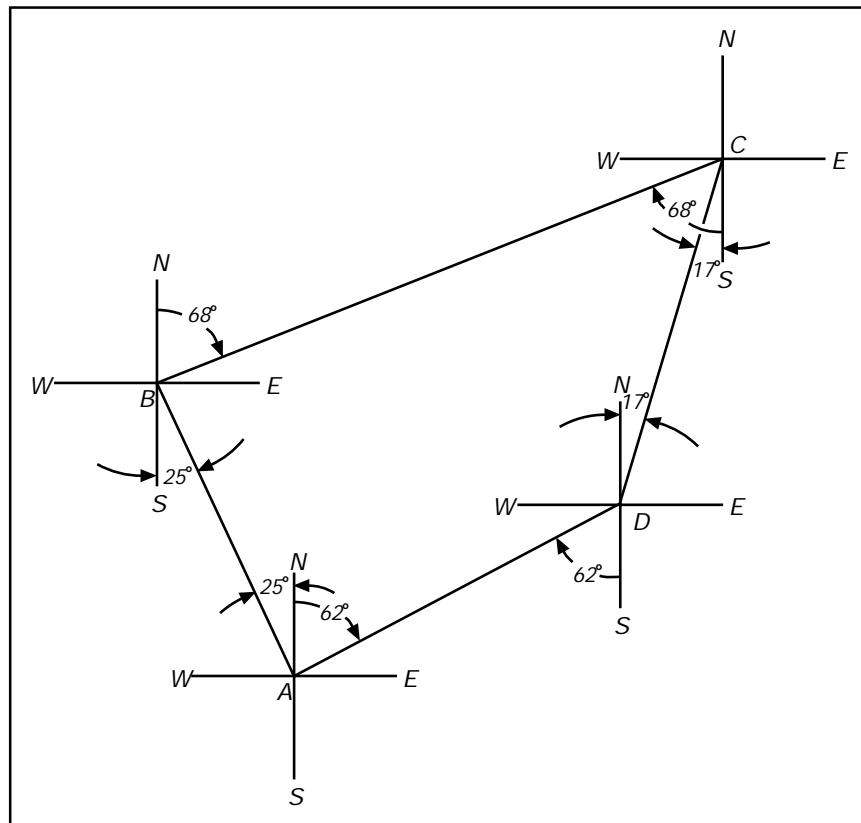
Figure 28

Line	Bearing
A – B	
B – C	
C – D	
D – A	

Relation between Bearing and Azimuths

To simplify computations based on survey data, bearings may be converted to azimuths or azimuths to bearings.

In the figure below, the first azimuth of 37° 30' is in the northeast quadrant since the angle eastward is less than 90°. In the northeast quadrant the bearing angle and the azimuth are identical.
The second azimuth, 112°45' is 112°45' from the north meridian. The bearing angle for this quadrant must be determined from the south meridian. Since the north and south meridian are 180° apart, one would subtract the azimuth, 112°45' from 180° to arrive at the bearing of 67°15'. Because it is in the southeast quadrant the bearing is S 67°15' E.

N.E. Quadrant: Bearing equals Azimuth

S.E. Quadrant: 180° - Azimuth = Bearing
    and   180° - Bearing = Azimuth

S.W. Quadrant: Azimuth - 180° = Bearing
    and   Bearing + 180° = Azimuth

N.W. Quadrant: 360° - Azimuth = Bearing
    and   360° - Bearing = Azimuth
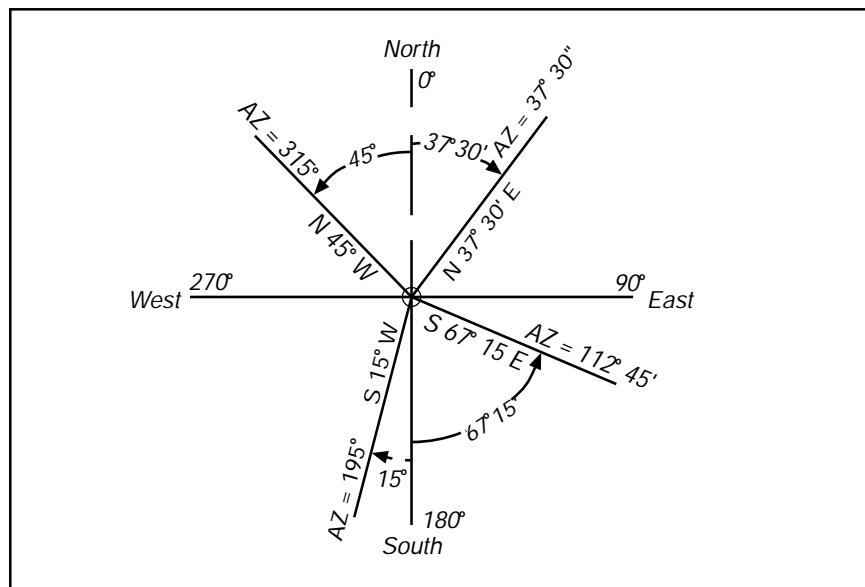
Figure 29

Back Azimuths and Back Bearings

The back azimuth or back bearing of a line is the azimuth or bearing of a line running in the reverse direction. The azimuth or bearing of a line in the direction in which a survey is progressing is called the forward azimuth or forward bearing. The azimuth or bearing of the line in the direction opposite to that of progress is called the back azimuth or back bearing.

The back azimuth can be obtained by adding 180° if the azimuth is less than 180° or by subtracting 180° if the azimuth is greater than 180°. The back bearing can be obtained from the forward bearing by changing the first letter from N to S or from S to N and the second letter from E to W or from W to E.

Bearing or Azimuth and Back Bearing or Back Azimuth
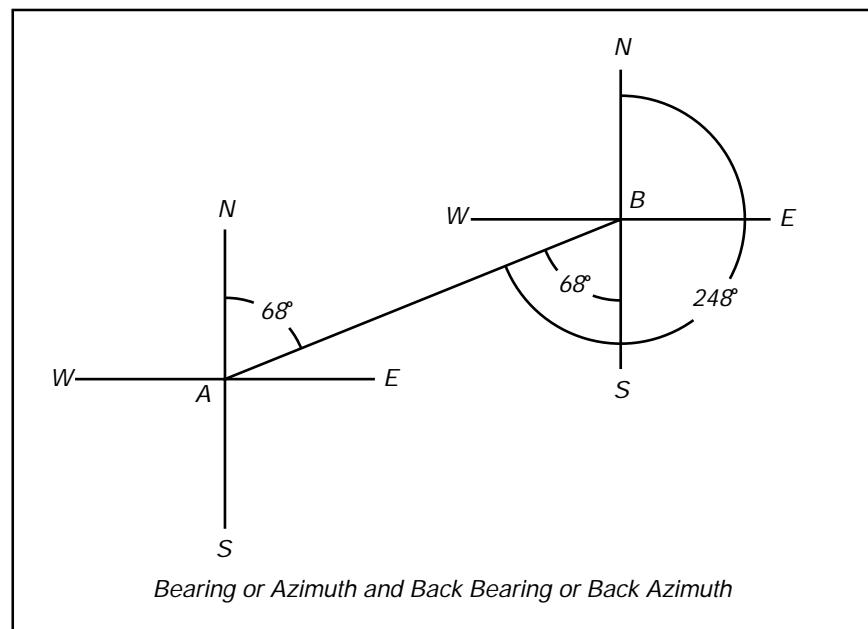
Figure 30

The bearing of the line A-B is N 68° E

The bearing of the line B-A is S 68° W.

The azimuth of the line A-B is 68°

The azimuth of the line B-A is 248°

Using angles to the right, calculate the bearings and azimuths of the lines.
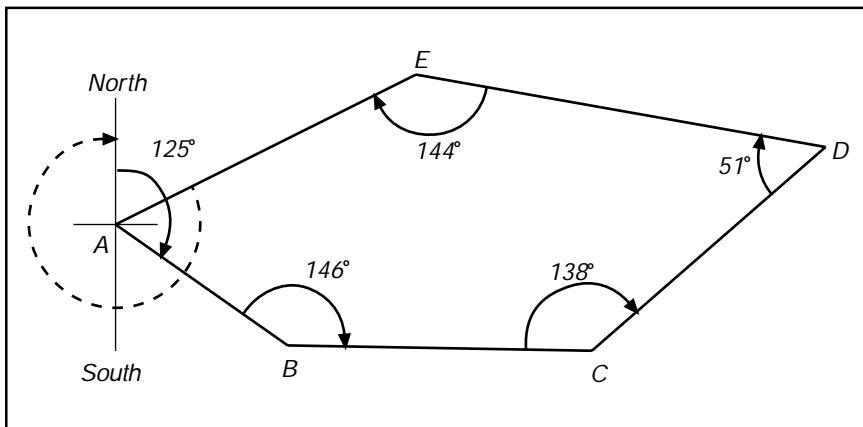
Figure 31

Line	Bearing	Azimuth
A – B		
B – C		
C – D		
D – E		
E – A		

Coordinates

In Surveying, one of the primary functions is to describe or establish the positions of points on the surface of the earth. One of the many ways to accomplish this is by using coordinates to provide an address for the point. Modern surveying techniques rely heavily on 3 dimensional coordinates.

In order to understand the somewhat complex coordinate systems used in surveying, we must first look at the Rectangular Coordinate System (or Cartesian Plane) from basic mathematics.

To keep it simple let's start by looking at a 1 dimensional system for locating points. Consider the horizontal line shown on the left of figure 32. A point on the line marked "0" is established as the origin. The line is graduated and numbered (positive to the right of the origin and negative to the left). Any number can be plotted on this line by its value and distances to other points on the line can be easily calculated. If all of our work was done precisely along a line, this system would be sufficient. We live in a 3 dimensional world, therefore we need a better system.

Let's look at a 2 dimensional system for locating points. The right of figure 32 shows a similar graduated line but in a vertical position. This line would function in a similar way as the horizontal line but giving locations of points in a different direction. By coinciding those lines at their respective origins we provide the foundation for a rectangular coordinate system.
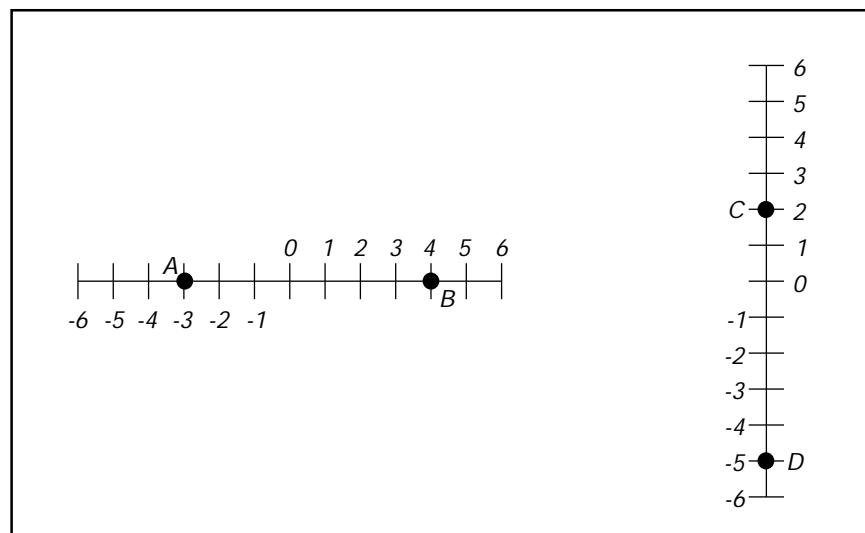
Figure 32

In the right of figure 33, is what is described as a *rectangular coordinate system.* A vertical directed line (*y*-axis) crosses the horizontal directed line (*x*-axis) at the origin point. This system uses an ordered pair of coordinates to locate a point. The coordinates are always expressed as (*x,y*).

The horizontal distance from the *y*-axis to a point is known as the *abscissa.* The vertical distance from the *x*-axis is known as the *ordinate.* The abscissa and ordinate are always measured from the axis to the point - never from the point to the axis.

The *x* and *y* axes divide the plane into four parts, numbered in a counter-clockwise direction as shown in the left of figure 33. Signs of the coordi-nates of points in each quadrant are also shown in this figure.

Note: In surveying, the quadrants are numbered clockwise starting with the upper right quadrant and the normal way of denoting coordinates (in the United States) is the opposite (*y,x*) or more appropriately *North, East.*
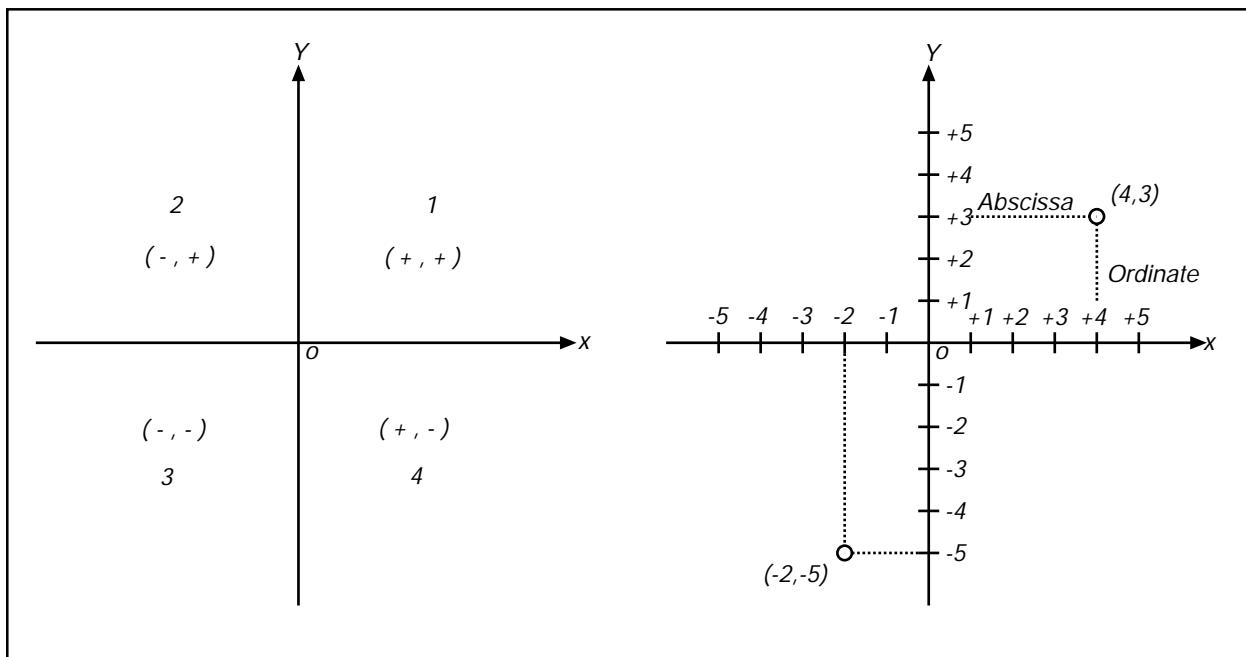
Figure 33

Determine the coordinates of the points shown in the figure below.
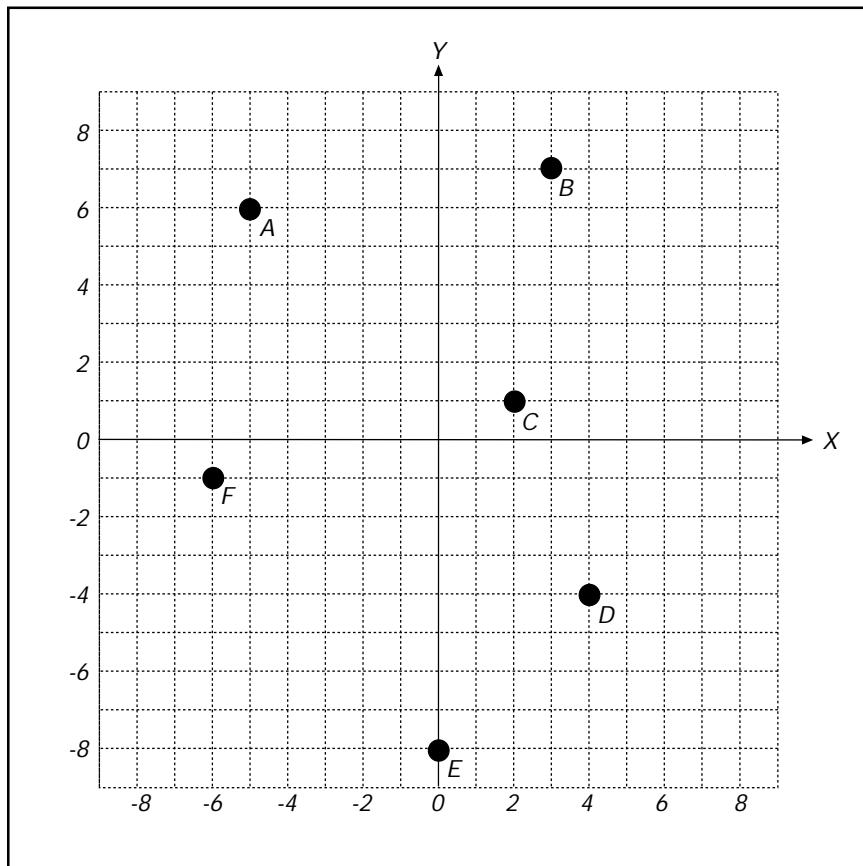
Point	X	Y	Point	N	E
A			D		
B			E		
C			F		

Polar Coordinates

Another way of describing the position of point *P* is by its distance *r* from a fixed point *O* and the angle θ that makes with a fixed indefinite line *oa* (the initial line). The ordered pair of numbers (*r*,θ) are called the polar coordinates of *P*. *r* is the radius vector of *P* and θ its vectorial angle.

Note: (*r*,θ), (*r*, θ + 360°), (-*r*, θ + 180°) represent the same point.

Transformation of Polar and Rectangular coordinates:

1. $x = r\cos\theta$ $y = r\sin\theta$ (if θ and *r* are known)

2. $r = \sqrt{x^2 + y^2}$ $\theta = \tan^{-1}\left(\frac{y}{x}\right)$ (if *x* and *y* are known)
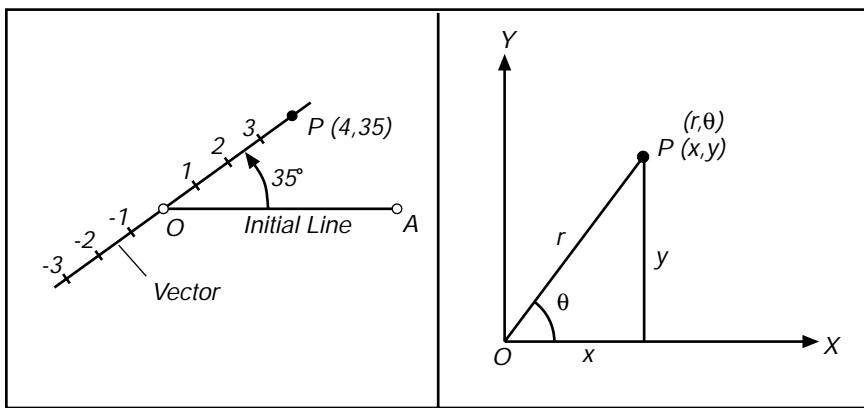
Figure 34

Measuring distance between coordinates

When determining the distance between any two points in a rectangular coordinate system, the pythagorean theorem may be used (see Review of Basic Trigonometry). In the figure below, the distance between *A* and *B* can be computed in the following way :

$$AB = \sqrt{[4\text{-}(\text{-}2)]^2 + [3\text{-}(\text{-}5)]^2} \qquad AB = \sqrt{[4+2)]^2 + [3+5)]^2} \qquad AB = 10$$

$$CB = +4\text{-}(\text{-}2) = 4+2 \qquad\qquad AC = +3\text{-}(\text{-}5) = 3+5$$

Point *C* in this figure was derived by passing a horizontal line through point *B* and a vertical line through point *A* thus forming an intersect at point *C*, and also forming a right triangle with line *AB* being the hypotenuse. The x-coordinate of *C* will be the same as the x-coordinate of *A* (4) and the y-coordinate of *C* will be the same y-coordinate of *B* (-5).
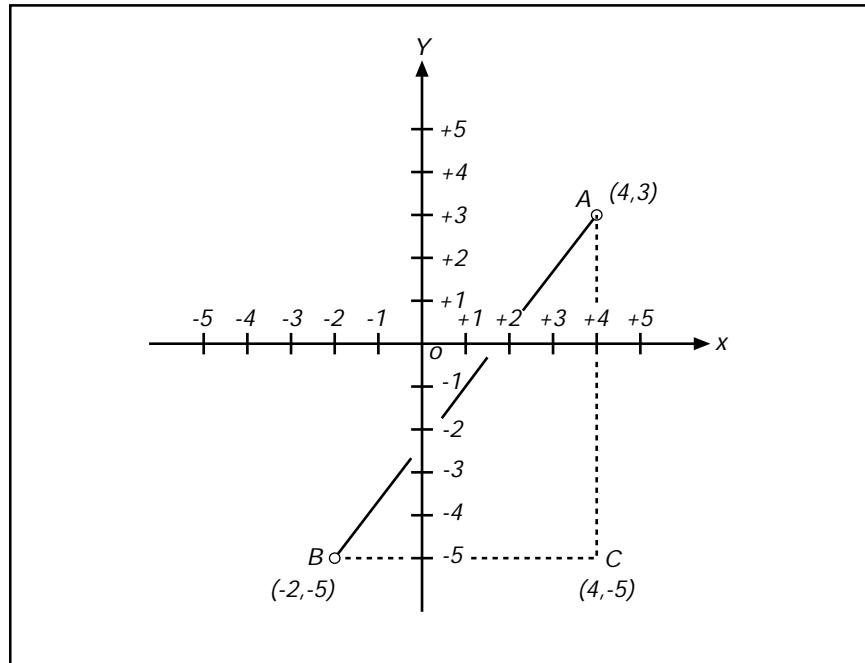
Figure 35

Inverse

In mathematics, the coordinates of a point are expressed as (*x,y*). In surveying, as mentioned earlier, the normal way of denoting coordinates (in the United States) is the opposite (*y,x*) or more appropriately *North, East.* The difference in Eastings between 2 points is referred to as the *departure* and the difference Northings is the *Latitude.*

To *inverse* between points means to calculate the bearing and distance between 2 points from their coordinate values.

Start by algebraically subtracting the Northings to get the Latitude, and the Eastings to get the Departure. A simple right triangle is formed and the pythagorean theorem can be used to solve for the hypotenuse (distance between points). To find the bearing we need to calculate the angle from the North/South line at one of the points by using basic trigonometry.
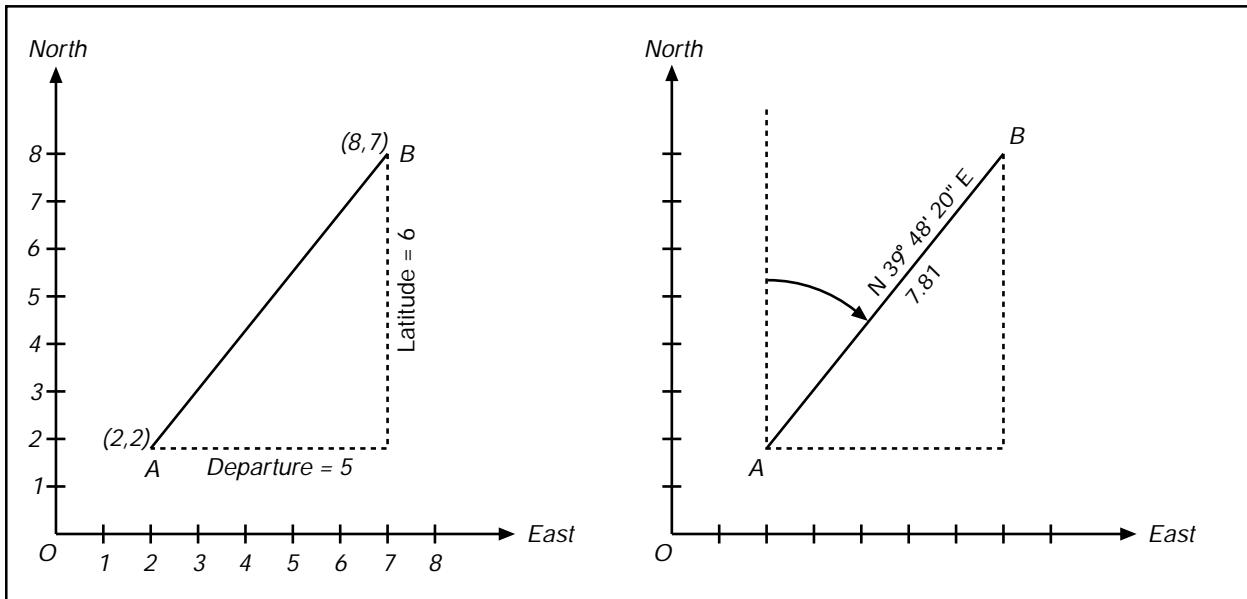
Figure 36

1. Plot the following points (N,E) and connect with lines in the following order ABCDEA.

   A (12,6)   B (-14,12)   C (-12,1)   D (-3,-9)   E (16,-10)

2. Find the bearing of each line (i.e. AB, BC, etc.) and the perimeter distance.

North

		20
		18
		16
		14
		12
		10
		8
		6
		4
		2

East

-2
-4
-6
-8
-10
-12
-14
-16
-18
-20

-14   -12   -10   -8   -6   -4   -2   0   2   4   6   8   10   12   14
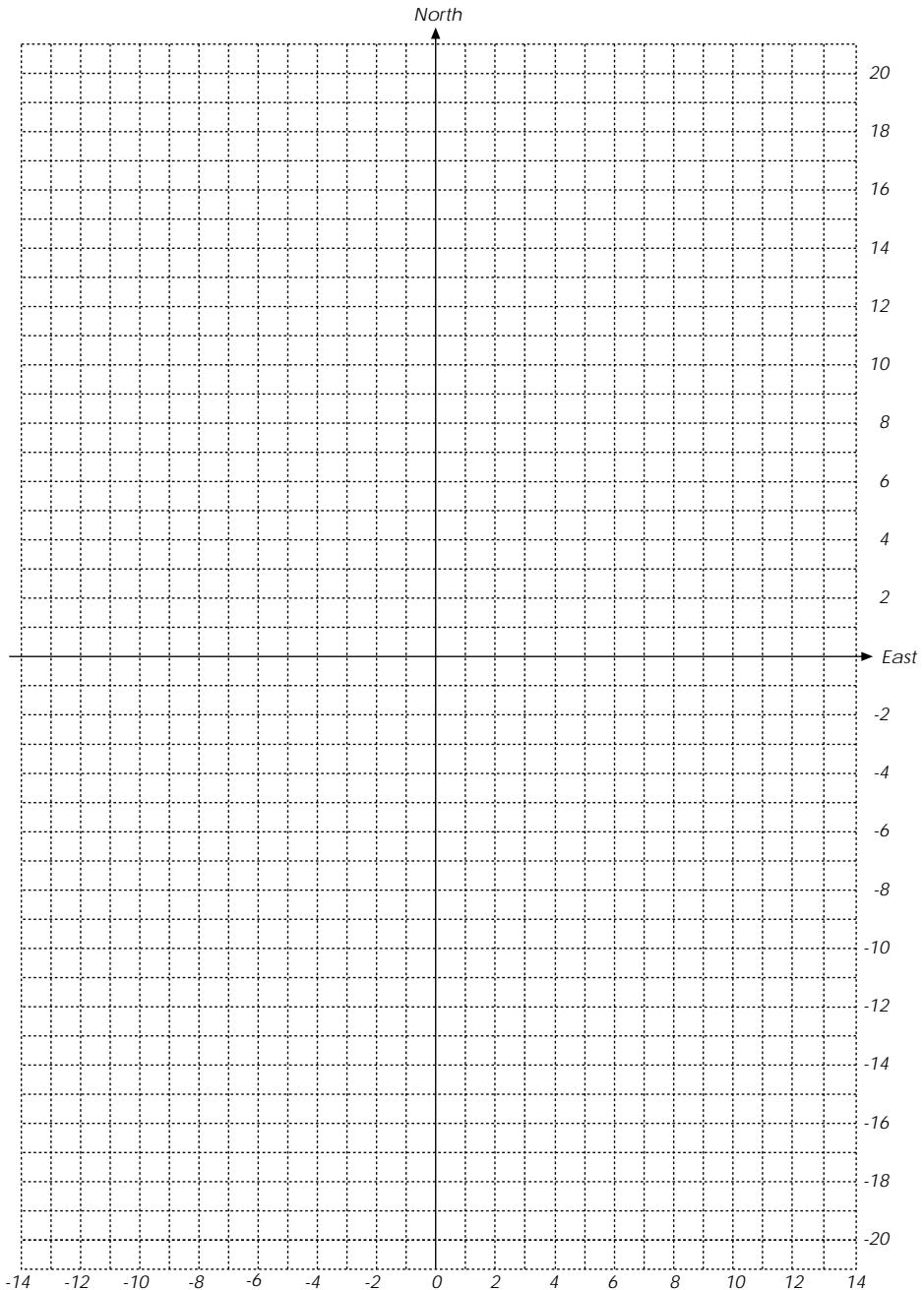
Area by Coordinates

Area of a trapezoid: one-half the sum of the bases times the altitude.

Area of a triangle: one-half the product of the base and the altitude.

The area enclosed within a figure can be computed by coordinates. This is done by forming trapezoids and determining their areas.

Trapezoids are formed by the abscissas of the corners. Ordinates at the corners provide the altitudes of the trapezoids. A sketch of the figure will aid in the computations.

This is similar to the double meridian distance method but does not use meridian distances. For land area calculations following a boundary traverse, the DMD method for area is more commonly used. The DMD method will not be discussed here.
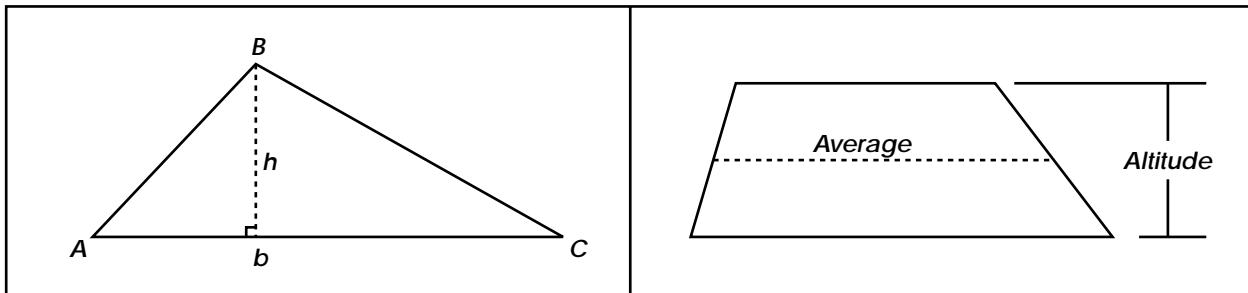
Figure 37

1. Find the latitude and departure between points.

2. Find the area of the figure.
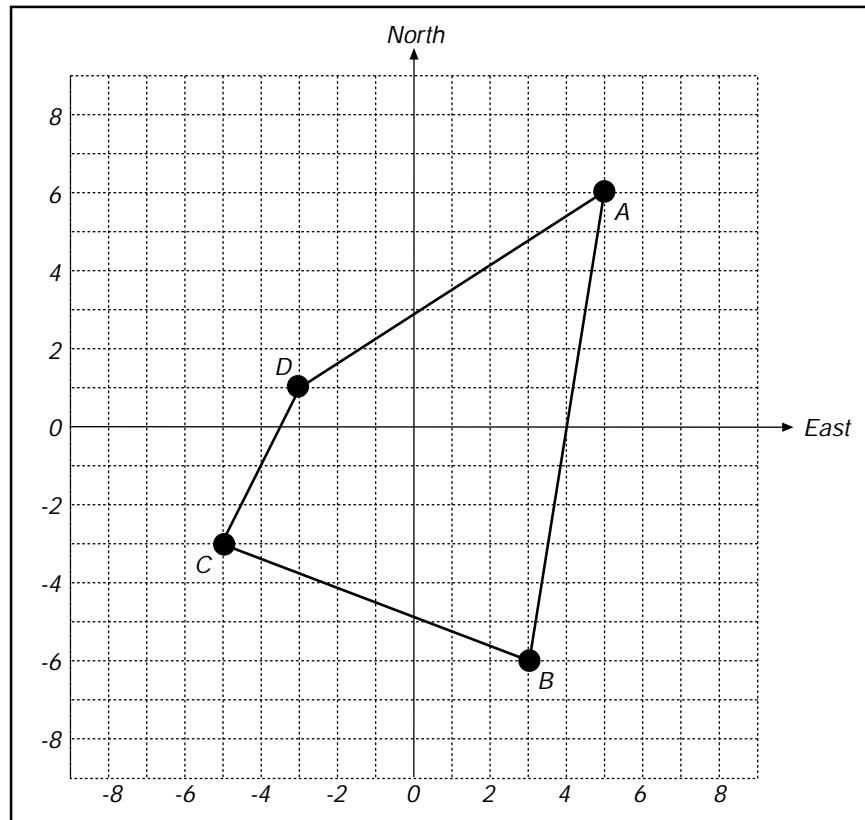
	Latitude	Departure
A-B		
B-C		
C-D		
D-A		

Area	

Complete the table below, then plot the points and lines.
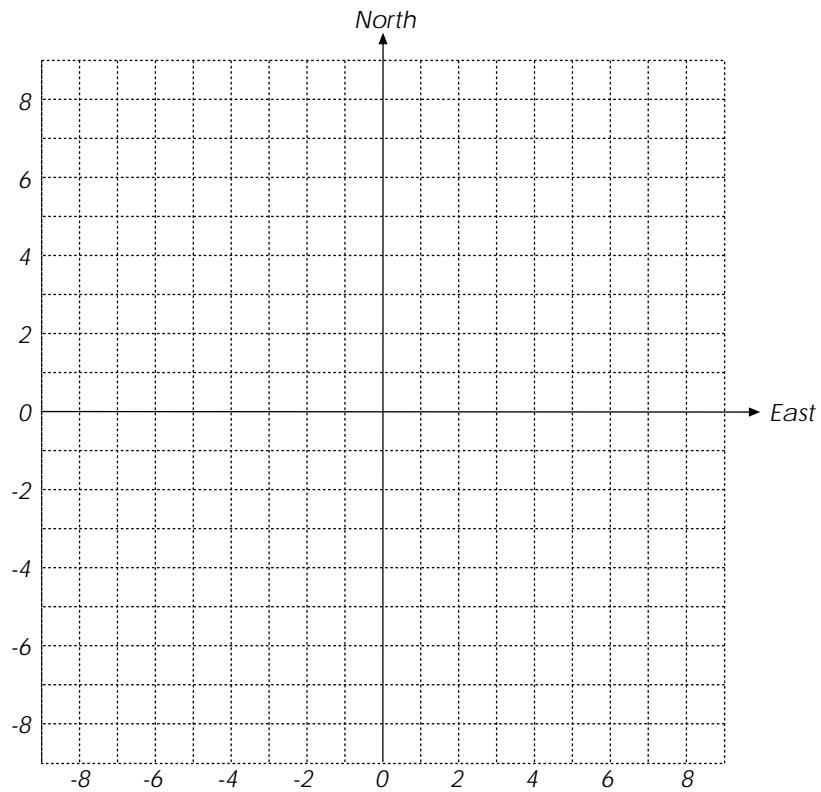
Point	Bearing	Distance	Latitude	Departure	Northing	Easting
A					8	8
			-16	-3		
B						
	N 74° 44′ 42″ W	11.40				
C						
			11	2		
D						
	N 80° 32′ 16″ E	12.17				
A						

Traverse

Definition of a traverse

A Traverse is a succession of straight lines along or through the area to be surveyed. The directions and lengths of these lines are determined by measurements taken in the field.

Purpose of a traverse

A traverse is currently the most common of several possible methods for establishing a series or network of monuments with known positions on the ground. Such monuments are referred to as *horizontal control points* and collectively, they comprise the *horizontal control* for the project.

In the past, triangulation networks have served as horizontal control for larger areas, sometimes covering several states. They have been replaced recently in many places by GPS networks. (GPS will be discussed in more detail later.) GPS and other methods capitalizing on new technology may eventually replace traversing as a primary means of establishing horizontal control. Meanwhile, most surveys covering relatively small areas will continue to rely on traverses.

Whatever method is employed to establish horizontal control, the result is to assign rectangular coordinates to each control point within the survey. This allows each point to be related to every other point with respect to distance and direction, as well as to permit areas to be calculated when needed.

Types of traverses

There are several types or designs of traverses that can be utilized on any given survey. The terms *open* and *closed* are used to describe certain characteristics of a traverse. If not specified, they are assumed to refer to the mathematical rather than geometrical properties of the traverse.

A *Geometrically Closed Traverse* creates a closed geometrical shape, such as the first two examples in Figure 38. The traverse ends on one of two points, either the on same point from which it began or on the initial backsight. The first two traverses in Figure 38 are geometrically closed.
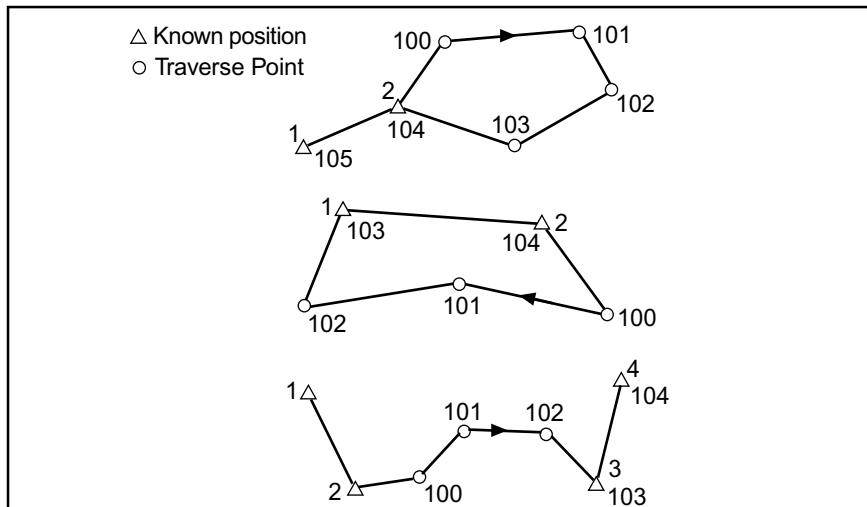
Figure 38

A *Geometrically Open Traverse* does not create a closed shape because it ends at some point other than the initially occupied point or the initial backsight. This type of traverse is sometimes expedient for the survey of a strip project such as a pipeline or highway. The third example in Figure 38 is a geometrically open traverse.

A *Mathematically Open Traverse* or simply an *Open Traverse* begins at a point of known position and ends at a point of previously unknown position. There is no method to verify that the measurements of the angles and distances are free from error. Consequently, this is not a desirable survey method.

A *Mathematically Closed Traverse* or simply a *Closed Traverse* begins at a point of known position and ends at a point of known position. Calculations can be made to check for errors. This method is preferred because the numbers can be confirmed. Figure 38 shows three different types of closed traverses.

One form of a closed traverse is a *"closed loop traverse"* which begins at a point of known position and ends at that same point. The first example in Figure 38 is a closed loop traverse. While the angles in this form of traverse can be checked for errors, no systematic error in the measuring device can be detected. Only blunders can be found.

To point this out lets consider an example of a closed loop traverse done with a transit and chain. The first time the traverse is run early in the morning on a cold day. No correction is used for the chain. The traverse is adjusted and meets the standards. Now the traverse is run again on a hot summer afternoon. No correction is used for the chain. Again the traverse is adjusted and meets the standards. Now when the coordinates of the first points are compared to the coordinates of the second points, we find that some of them are not close to one another . This is particularly true of those that are the farthest from the beginning of the traverse. The problem was that the chain was not the same length due to temperature changes, and this type of traverse will not show this type of error.

In a closed loop traverse, or on any geometrically closed traverse, there is also no check on the "basis of bearing."

This is an acceptable traverse method but care should be taken that the distance measuring equipment is properly calibrated and that the basis of bearing is correct.

Procedure for running a traverse

To begin any traverse, a **known point** must be *occupied.* (To occupy a point means to set up and level the transit or theodolite, directly over a monument on the ground representing that point.) Next, a direction must be established. This can be done by sighting with the instrument a second known point, or any definite object, which is in a known direction from the occupied point. The object that the instrument is pointed to in order to establish a direction is known as a *backsight.* Possible examples would be another monument on the ground, a radio tower or water tank on a distant hill, or anything with a known direction from the occupied point. A celestial body such as Polaris or the sun could also be used to establish an initial direction.

Once the instrument is occupying a known point, for example point number 2, and the telescope has been pointed toward the backsight, perhaps toward point number 1, then an angle and a distance is measured to the first unknown point. An unknown point being measured to is called a *foresight.* With this data, the position of this point (lets call it point number 100) can be determined. In Figure 38, there are graphical representations of three sample traverses, each beginning with the process described here.

The next step is to move the instrument ahead to the former foresight and duplicate the entire process. The former occupied point becomes the backsight and a new unknown point becomes the foresight. This procedure is repeated at each point until measurements have been taken to all the needed points.

Calculating coordinates for traverse

To calculate the coordinates for each point on a traverse, the direction and distance from a known point must be also be known. Typically, the distance is measured in the field, but the direction is not. It must be computed from the angles measured in the field. The specific procedure will vary depending on the type of field angles measured and whether bearings or azimuths are used to describe directions. Refer to the section of this manual on "Bearings and Azimuths" for more detail. It is also helpful to draw a sketch of each angle to help visualize what is happening.

Once the distance and direction are known, the latitude and departure can be calculated using right triangle trigonometry as discussed in the previous section on "Coordinates." These values will indicate the distances north or south and east or west between the two points. The coordinates on the unknown point can then be determined by algebraically adding the latitude to the northing of the known point and the departure to the easting of the known point. A positive, or north, latitude is added to the northing while a negative, or south, latitude (or the absolute value of the latitude) is subtracted from the northing. A positive, or east, departure is added to the easting while a negative, or west, departure (or the absolute value of the departure) is subtracted from the easting.

These calculations are repeated for each point along the traverse. The coordinates of the last point are used as a base for each new point. The new latitude and departure are added to this base. The last point of a (mathematically) closed traverse is designated as the closing point. If the traverse is a (mathematically) closed traverse, the calculated coordinates for the closing point should be equal to or nearly equal to the record or previously known coordinate values for that point.

Precision of the traverse

The odds having exact closure using the raw angles and distances from a traverse are astronomical. There will usually be some discrepancy between the record coordinates and those calculated in the traverse. By inversing between the two sets of coordinates, the linear distance between them can be computed. This value is called the *linear closure* for the traverse and gives us an idea of how much error there was in the field measurements. A small error is most likely due to the limitations on how precisely the angles and distances can be measured with the specific equipment. A large error would indicate that there is a blunder in the measurements.

The linear closure suggests how well the measuring was done. But while a half foot might seem like a small error when measuring ten miles, it would seem a rather large error when cutting an eight-foot board. To better determine whether the error in a particular traverse is acceptable or not, we compare it to the distance traversed. This comparison is frequently called *precision*, and gives us a much better way to evaluate the error.

The Precision of a traverse is expressed as the ratio of the "linear error of closure" to the "traverse perimeter", and is called the *"closure ratio."* A traverse that is 12,000 feet in length and does not close by 1 foot, is said to have "1 in 12,000 closure." If that same traverse does not close by 0.10 feet, then it has 1 in 120,000 closure.

Balancing a traverse

Balancing the traverse is a procedure that distributes and apportions field measurement errors. Adjustments are made to give the traverse an exact closure. In a closed-loop traverse the Northings and the Southings must be equal to each other and the Eastings must be equal to the Westings. There are several methods for balancing coordinates.

The *Transit Rule* is used when angular measurement is much better than distance measurement. This procedure was developed for chain and transit surveys where angles could be measured rather precisely. The distances, on the other hand, were subject to all of the possible errors of break and slope chaining over rough terrain. It distributes the traverse error in latitude proportionally to the latitude of the individual legs. Traverse legs with the largest change in latitude absorb the most error in latitude. The traverse error in departure is distributed in proportion to the departure of the individual legs.

The *Crandall Method* is also used when angular measurement is much better than distance measurement. It also distributes the error in a comparable way.

The *Compass Rule* is used when the accuracy of angular measurement is about equal to the accuracy of distance measurement. This procedure was developed during the days of surveying with a chain and staff compass. It is a two-step process. First, the angular error is meted out to provide angular closure. Secondly, the coordinates are developed using the adjusted angles and then the coordinates are adjusted. It distributes the traverse error in proportion to the length of the individual legs. Although this method was designed for some early, low precision equipment, this method works well for an EDM-theodolite traverse. The assumption that the precision of angles and distances is comparable generally holds true.

The *Least Squares Method* works well whatever the relative accuracy of angular or distance measurements. Weighted values can be given for each measurement. This causes more reliable measurements to be given more influence in the adjustment. This procedure also does not require a single line traverse. The Least Squares Method can be applied to complex networks and traverses with measurements between multiple points. This allows more flexibility in establishing control and greater opportunities for locating blunders. Error distribution is similar to the compass rule. The calculations are much more complex and demand a computer. The Federal Geodetic Control Committee cites this method in their standards.

Balancing a traverse by Compass Rule

Balancing by the Compass Rule is a two-step process. First, the angular error is meted out to provide angular closure. This is done by computing a direction on the closing course of the traverse and comparing it to the record direction. Any difference between the two is divided by the number of angles measured and each angle modified by that amount. Care needs to be taken, particularly if types of angles measured were not consistent, that each angle is modified in the right direction. The final course should now agree and the direction of each coarse should have changed by a prorated amount.

If certain conditions are met, the angular error can be determined using the formula shown in Figure 39. The traverse must be geometrically closed as shown at the top of Figure 39. In addition, all angles must be measured in the same direction, either as internal angles or as external angles. This method will still work on the traverse in the bottom of Figure 39. However, care should be taken that only the required angle is counted at the initial point of the traverse, regardless of how many angles were actually measured.
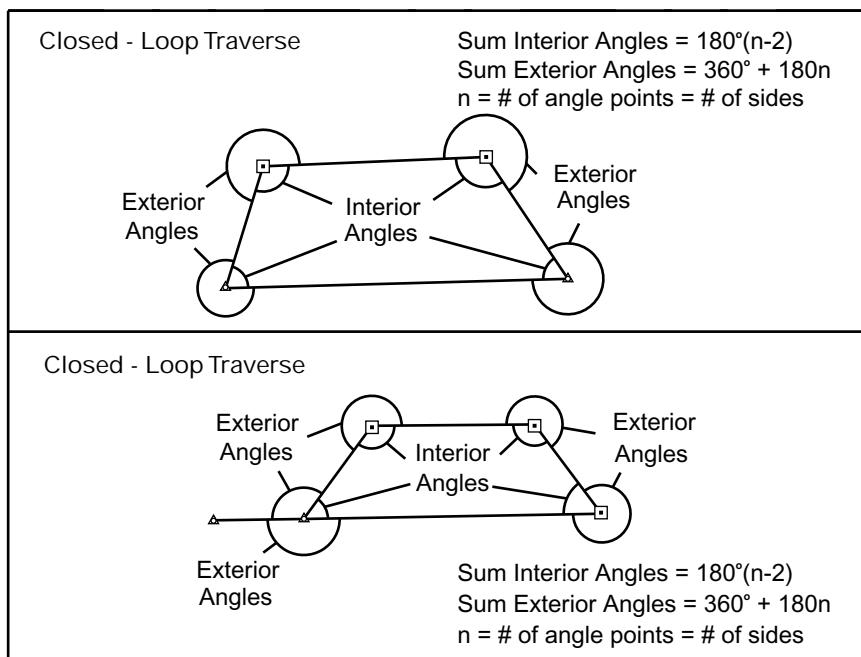
Figure 39

Calculate and balance the traverse azimuths below using the compass rule.

Point I.D.	Measured Angle	Measured Azimuth	Azimuth Adjustment	Adjusted Azimuth
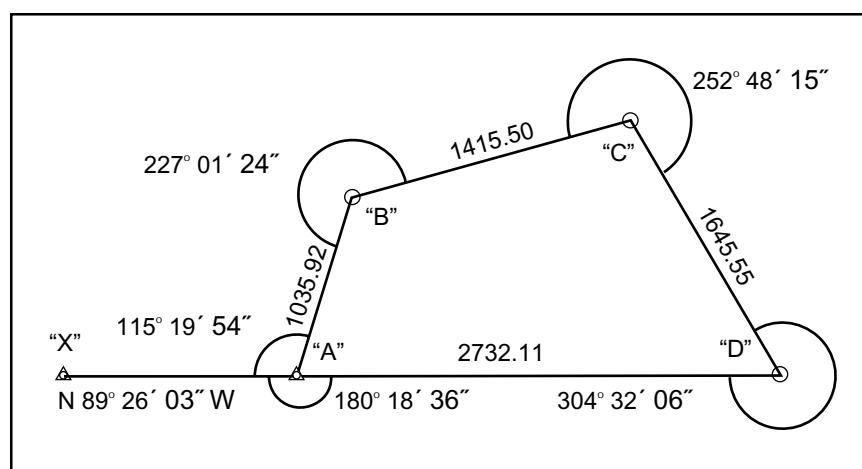
Next, the coordinates for each point are developed and recorded using the adjusted angles. Then the errors in latitude and departure are distributed in proportion to the lengths of the traverse legs. The longest leg of the traverse will receive the most adjustment in both latitude and departure. The shortest leg will receive the least.

The latitude adjustment for any one leg is:

$$(Latitude\ Adjustment) = \frac{(Total\ Latitude\ Adjustment)}{(Traverse\ Perimeter)}\ (Length\ Of\ The\ Leg)$$

Similarly, the departure adjustment for any one leg is:

$$(Departure\ Adjustment) = \frac{(Total\ Departure\ Adjustment)}{(Traverse\ Perimeter)}\ (Length\ Of\ The\ Leg)$$

The adjustments for both latitude and departure are computed and recorded for each course. The final adjusted latitudes and departures are determined by algebraically adding the corrections to the latitudes and departures that were calculated from the adjusted angles.

To complete the process, the final adjusted coordinates are calculated by algebraically adding the latitude and departure for each point to the coordinates of the prior point. The closing point should have the same coordinates as record.

Calculate and balance the traverse coordinates using the compass rule.

Point I.D.	Adjusted Azimuth	Horizontal Distance	Latitude	Departure	Adjustment		Coordinates	
					Latitude	Departure	Northing	Easting

The Global Positioning System

The Global Positioning System *(GPS)* is a navigational or positioning system developed by the United States Department of Defense. It was designed as a fast positioning system for 24 hour a day, three dimensional coverage worldwide.

It is based on a constellation of 21 active and 3 spare satellites orbiting 10,900 miles above the earth. The GPS *(NAVSTAR)* satellites have an orbital period of 12 hours and are not in geosynchronous orbit (they are not stationary over a point on the earth). They maintain a very precise orbit and their position is known at any given moment in time. This constellation could allow a GPS user access to up to a maximum of 8 satellites anywhere in the world.

GPS provides *Point Position* (Latitude/Longitude) and *Relative Position* (Vector). GPS can differentiate between every square meter on the earth's surface thus allowing a new international standard for defining locations and directions.

The applications (military or civilian) for GPS are almost limitless, from guiding a missile to a target with incredible accuracy or tracking and monitoring the location of a city's emergency vehicles or providing a zero visibility landing and air collision avoidance system to a variety of surveying applications.

The Basic Principles of GPS

For centuries man has used the stars to determine his position. The extreme distance from the stars made them look the same from different
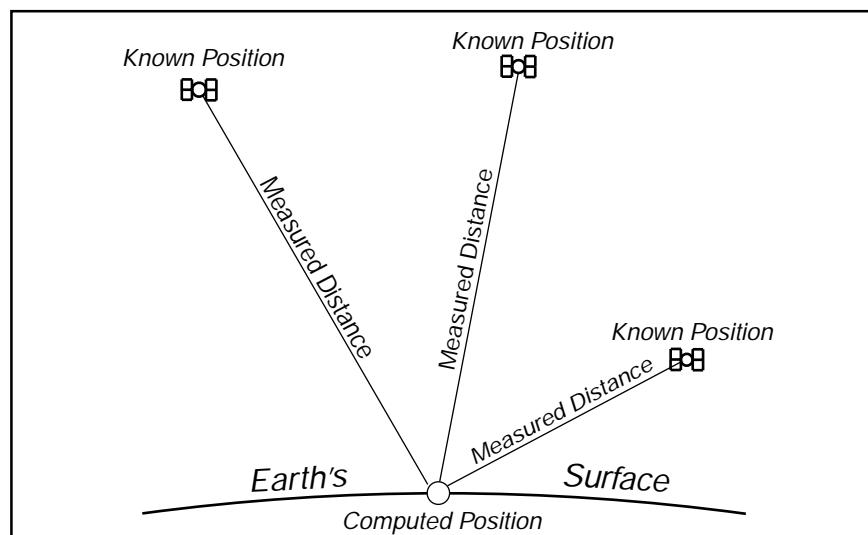
Figure 40

locations and even with the most sophisticated instruments could not produce a position closer then a mile or two. The GPS system is a constellation of *Manmade Stars* at an orbit high enough to allow a field of view of several satellites, yet low enough to detect a change in the geometry even if you moved a few feet.

A typical conventional survey establishes positions of unknown points by occupying a known point and measuring to the unknown points. GPS is somewhat the opposite. We occupy the unknown point and measure to known points. In conventional surveying this is similar to the process of doing a resection, the slight difference is that the targets are 10,900 miles away and travelling at extremely high speeds!

How positions are computed

Think of the satellites as precise reference points for our survey and we are using satellite ranging to trilaterate our position (trilateration is the process of determining positions by measuring the lengths of triangles, while triangulation is the process of determining the positions by measuring the angles of triangles). If we know the distance from one satellite our position could be anywhere on the surface of an imaginary sphere with the satellite at the center of that sphere. This obviously does not give us our position. If we knew the distance to a second satellite our position could be anywhere on the circle formed by the intersection of the two imaginary spheres with the satellites at their centers. This still does not give us a single position. If we include the distance to a third satellite into our scenario, we find that by intersecting three spheres, two solutions exist. Usually one of these solutions yields a position nowhere near the earth and can be discarded. If you know the elevation of your position, you can eliminate the need for one satellite. One of the spheres in the computation would be the earth with a radius of the earth plus your elevation.

How satellite distance is measured

Each GPS satellite continually broadcasts a radio signal. Radio waves travel at the speed of light (186,000 miles per second) and if we measure how long it took for the signal to reach us we could compute the distance by multiplying the time in seconds by 186,000 miles per second.

In order to measure the travel time of the radio signal, the satellite broadcasts a very complicated digital code. The receiver on the ground generates the same code at the exact time and when the signal is received from the satellite, the receiver compares the two and measures the phase shift to determine the time difference.
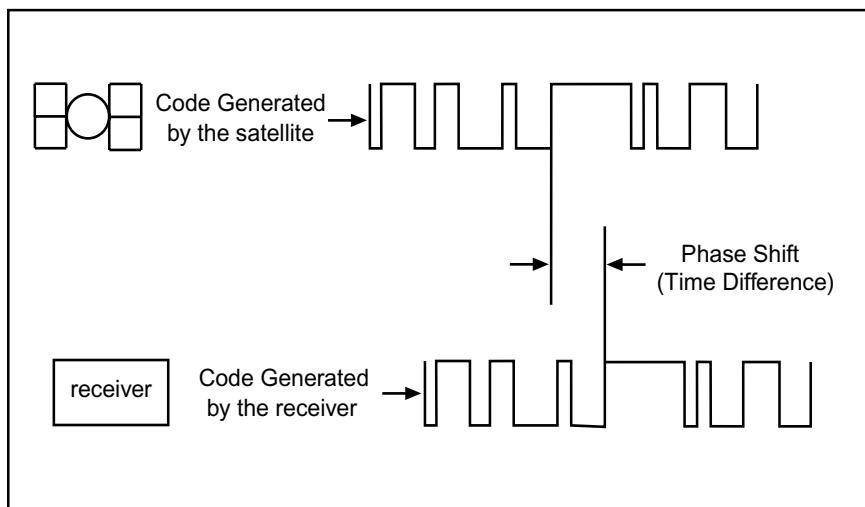
Figure 41

If the satellites are orbiting about 10,900 miles above the surface of the earth and the radio signal travels at 186,000 miles per second, a satellite directly above takes about 0.06 seconds to transmit its signal to earth. To be able to measure the distance to the accuracy needed, the satellite and the receiver must be perfectly in sync and we must have the ability to measure time with extreme accuracy. Each satellite is equipped with 4 atomic clocks which keep almost perfect time and the receivers that we use can measure time to an accuracy of 0.000000001 of a second. The problem is that the receivers are not in sync with the atomic clocks. If the receiver is out of sync with the satellite by even 0.001 of a second, the computed distance would be off by 186 miles!

To solve this problem we include the measurement to an extra satellite. The added sphere to the equation would intersect at the same point as before *if the receiver were in perfect sync with the satellite.* If the added sphere does not intersect at the same point, then the clocks are not in perfect sync and a clock offset for the receiver can be calculated. For accurate 3D positions, 4 satellites must be in view of the receiver.

The Ephemeris

Once we know the distance to the satellite, we need to know exactly where the satellite was at the moment of the measurement. Receivers have an almanac stored in their memory which gives each satellite's position in the sky at any given time. Contradicting what was mentioned earlier, the satellite's orbit does decay changing its position, altitude and speed. This change is extremely minuscule and is monitored by the Department of Defense every 12 hours and these variations (ephemeris errors) are transmitted back to the satellite. The satellite transmits a data message along with its pseudo-random code. The data message contains information about its exact orbital location (with the ephemeris error corrections) and its system's health.

Sources of error and expected GPS accuracy

GPS positions are affected by various errors such as the clock and satellite position errors mentioned earlier. Other errors include: ionospheric and atmospheric errors which are caused by the slowing down the radio waves as they travel through the ionosphere and lower atmosphere; multipath errors which are caused by the radio waves bouncing off objects before reaching the receiver; and poor geometry in the satellite positions. In addition to all these errors there is a deliberately caused error called Selective Availability (SA) which is the degrading of the radio signal by the Department of Defense. This manmade error can be turned on as needed to deny hostile forces the advantage of GPS positioning.

The ultimate accuracy of GPS positions are determined by the sum of all these errors. It is difficult to quantify this specification as receiver manufacturers are constantly finding new ways to improve accuracy. There are a variety of receivers on the market yielding a variety of levels of accuracy. Receivers typically fall into 3 categories; Survey, mapping, and navigation grades. Currently, survey grade receivers can achieve accuracies in the millimeter range, mapping grade receivers through the use of post-processing or real time correction using a base station can achieve 1 to 3 meter accuracy, and navigational receivers can achieve 5 to 12 meter accuracy (with SA off).

Differential GPS

To achieve sub-centimeter accuracies in positions, we need a survey grade receiver and a technique called *Differential GPS.* By placing a receiver at a known location, a total error factor which accounts for all the possible errors in the system, can be computed which can be applied to the position data of the other receivers in the same locale. The satellites are so high-up that any errors measured by one receiver could be considered to be exactly the same for all others in the immediate area.
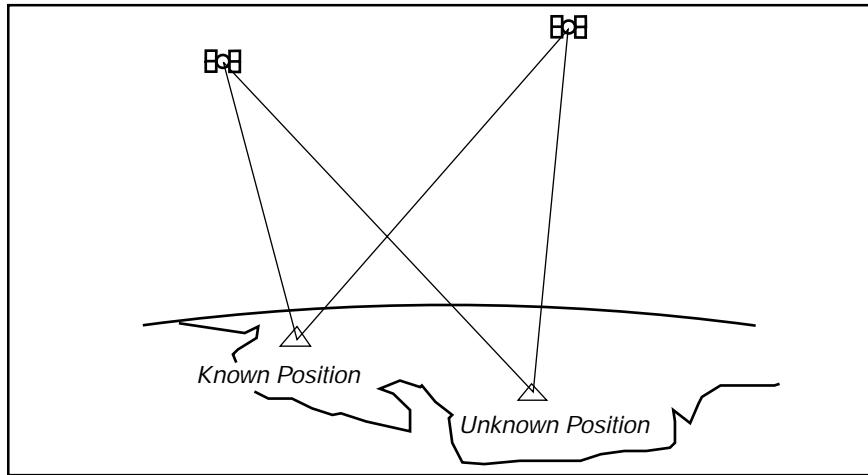
Figure 42

Differential Leveling

Differential leveling is the process used to determine a difference in elevation between two points. A Level is an instrument with a telescope that can be leveled with a spirit bubble. The optical line of sight forms a horizontal plane, which is at the same elevation as the telescope crosshair. By reading a graduated rod held vertically on a point of known elevation (Bench Mark) a difference in elevation can be measured and a height of instrument (H.I.) calculated by adding the rod reading to the elevation of the bench mark. Once the height of instrument is established, rod readings can be taken on subsequent points and their elevations calculated by simply subtracting the readings from the height of instrument.
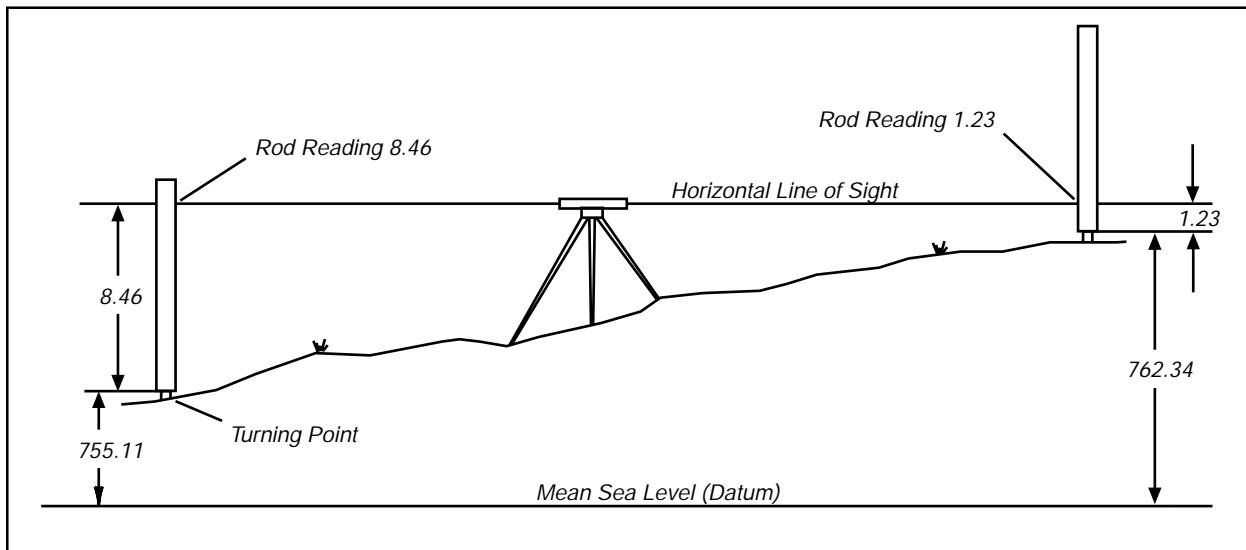
Figure 43

Running a line of levels

In the following example, the elevation at BM-A is known, and we need to know the elevation of BM-K. The level is set up at a point near BM-A, and a rod reading taken. The height of instrument (HI) is calculated and a rod reading to a turning point (TP1) is taken. The reading of the foresight is subtracted from the height of instrument to obtain the elevation at TP1. The rod stays at TP1, the level moves ahead and the rod at TP1 now becomes the backsight. This procedure is repeated until the final foresight to BM-K.
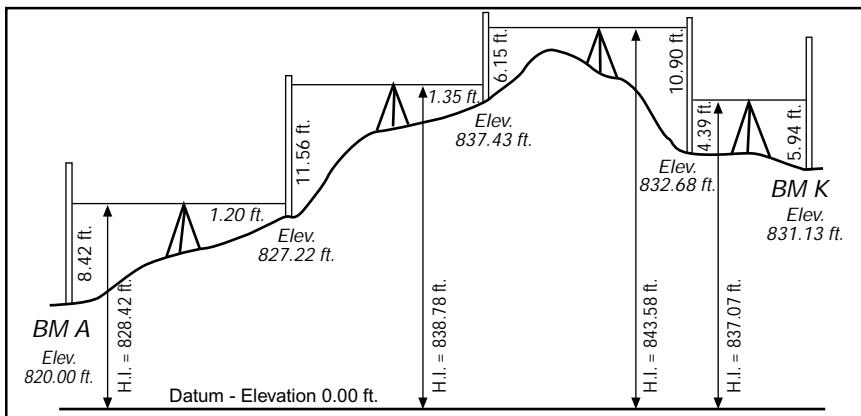
Figure 44

Sta	BS (+)	H.I.	FS (-)	Elev	Description
BM A	8.42			820.00	BM A: Top of Iron Pipe, 3" diameter
		828.42			at corner of Wishburn and Oak Dr.
TP 1	11.56		1.20	827.22	
		838.78			
TP 2	6.15		1.35	837.43	
		843.58			
TP 3	4.39		10.90	832.68	
		837.07			
BM K			5.94	831.13	BM K: Top of iron pipe, 2" diameter
					Corner of Wishburn and Oxford.
BS Sum = 30.52		FS Sum = 19.39			CHECK: Begin Elevation = 820.00
					BS/FS Difference = 11.13
Difference = 30.52 - 19.39 = 11.13					Ending Elevation = 831.13

Closing the Level Loop

A level loop is closed either to another Benchmark or back to the starting Benchmark. To check for errors in the loop sum the Backsights (BS) and the Foresights (FS). Calculate the difference between the BS and FS (BS-FS). Algebraically add this difference to the starting elevation, to yield the closing elevation. This elevation should be within accepted industry standards tolerances of the closing Benchmark's published elevation.

Instrumental Errors

The most common instrumental error is caused by the level being out of adjustment. As has been previously stated, the line of sight of the telescope is horizontal when the bubble is in the center of the tube, provided the instrument is in perfect adjustment. When it is not in adjustment, the line of sight will either slope upward or downward when the bubble is brought to the center of the tube.

Instrumental errors can be eliminated if kept at a minimum by testing the level frequently and adjusting it when necessary. Such errors can also be eliminated by keeping the lengths of the sights for the backsight and foresight readings nearly equal at each setting of the level. Since it is never known just when an instrument goes out of adjustment, this latter method is the more certain and should always be used for careful leveling.

Extremely long sights should also be avoided. The further the rod is from the level, the greater the space covered on the rod by the cross hair and the more difficult it will be to determine the reading accurately. For accurate results, sights with the engineer's level should be limited to about 300 feet.

Complete the following level circuit, compute misclosure.

Sta	BS (+)	H.I.	FS (-)	Elev	Description
BM X	6.72			935.42	BM X: Brass Disk in walk
					N.W. corner 12th and Sunset
TP 1	7.13		2.18		Elev: 935.42
TP 2	4.19		3.23		
TP 3	6.72		5.11		
TP 4	1.09		7.23		
BM Y			2.36		BM Y: 1/2" Iron Rod
					S.E. Corner 18th and Sunrise
					Elev: 941.19

Trigonometric Leveling

This leveling procedure involves observing the vertical (or zenith) angle and slope distance between two points. The difference in elevation can then be calculated. Within the limits of ordinary practice, triangle BEC (figure 45) can be assumed to be a right triangle and:

EC = BC x cos(zenith angle)

A major source of error in determining the difference in elevation by this method is the uncertainty in the curvature and refraction caused by variations in the atmospheric conditions.
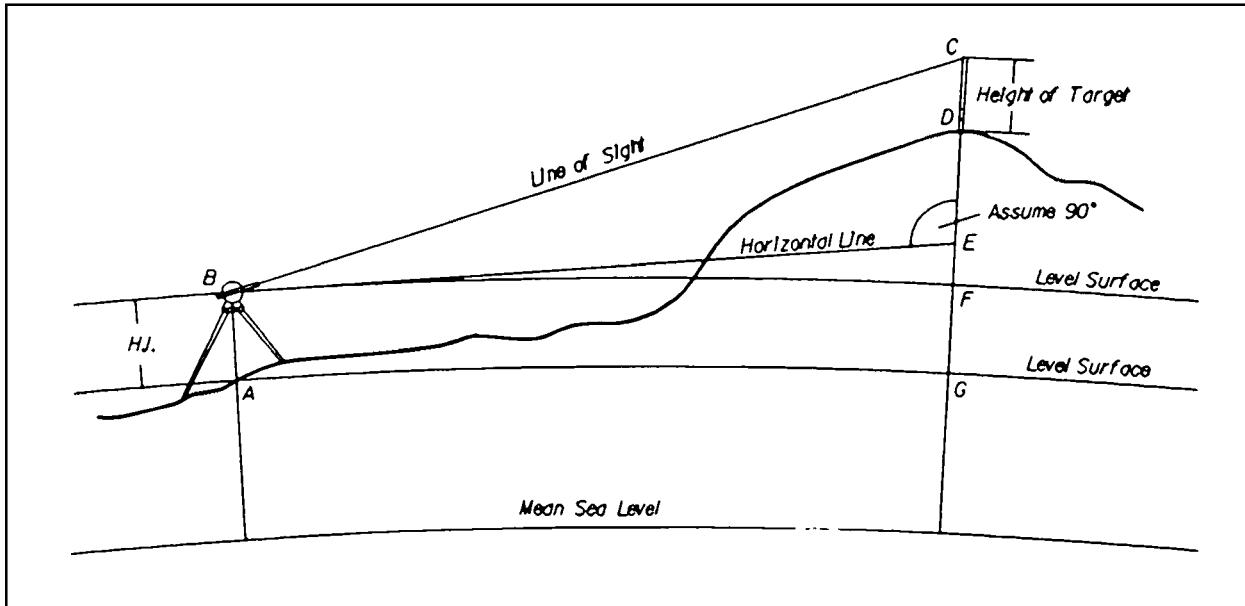
Figure 45

The effects of Earth Curvature and Atmospheric Refraction must be taken into account when using trigonometric methods to determine elevations. A line of sight perpendicular to a plumb line lies in a horizontal plane. The earths curved surface departs from this line by the value c (shown in Figure 45, as the distance E-F).

For most surveys, a practical value for curvature is:

$c = 0.667M^2$

Where M is the sight distance in Miles
and c is the earths curvature in Feet.

Due to the density of the air, the optical line of sight refracts or bends back towards the earth, negating about 14% of the effects of curvature.

The combined effect of Curvature and Refraction is:

$(c+r) = 0.574M^2$

There are two acceptable methods to correct for Curvature and Refraction if the formulas are not applied:

Balance the Backsights and Foresights

Observe the zenith angles from both ends of the line (reciprocal zeniths).

The effects of Curvature and Refraction increases rapidly with distance as shown in the table below:

Distance	Effects of Curvature and Refraction				
	200 ft	500 ft	1000 ft	1 mile	2 mile
(h) feet	0.001	0.005	0.021	0.574	2.296

When using trigonometric methods to establish accurate elevations, the following must be taken into consideration:

- Due to the effects of curvature and refraction, the instrument to target distance must be kept relatively short. A good rule of thumb is not to exceed 1000 feet.

- Make sure you understand your equipment's capabilities. Instruments that can measure zenith angles and slope distances to a high order of accuracy will produce good trigonometric elevations.

- Setup and level your instrument and target carefully. Measure the height of instrument and height of target accurately.

- Measure several slope distances and use a representative or mean value. Make sure that your EDM is correcting for the appropriate atmospheric conditions.

- Measure Direct and Reverse zenith angles, and use the adjusted value for your calculations.

- For lines longer than 500 feet, correct for curvature and refraction.

Modern Total Station instruments have built in capabilities to reduce and display trigonometric elevations.

Refer to figure 45, for the following exercise.

Given the following:

Elevation of Point A	506.78 ft
Height of Instrument	5.21 ft
Height of Target	5.46 ft
Measured Slope Distance	837.58 ft
Direct Zenith Angle	78°37'42"
Reverse Zenith Angle	281°22'28"

Correcting for curvature and refraction, calculate the elevation of point D.

Cross Sections

12

Cross sections are lines 90 degrees perpendicular to the alignment (P-Line, L-Line, centerline of stream, etc.), along which the configuration of the ground is determined by obtaining elevations of points at known distances from the alignment.

Cross sections are used to determine the shape of the ground surface through the alignment corridor. The shape of the ground surface helps the designer pick his horizontal and vertical profile. Once the alignment is picked, earthwork quantities can be calculated. The earthwork quantities will then be used to help evaluate the alignment choice.

In addition to earthwork calculations, cross sections are used in the design of storm sewers, culvert extensions and the size and location of new culverts. Because of this fact it becomes more important to get the additional sections at the points of interest that do not fall on the 50 foot stations.

The traditional method of taking cross sections starts with an alignment staked out in the field. A profile is run over the centerline stations by differential leveling. Cross section lines are laid out 90 degrees to the alignment, often with a right angle prism. Usually elevations are determined with an engineer's level and rod in level terrain or with a hand level and rod in rough, irregular country. For each cross section, the height of instrument is determined by a backsight on the centerline station. The rod is then held on the cross section line at breaks in the surface slope, where rod readings are observed and distances measured with a tape. Cross sections are usually taken at even stations and points of interest or irregularity along the alignment.

An example of cross section notes:

1. Stationing runs from the bottom of the page to the top.

2. Notes are taken looking ahead on line.

3. Record all topographical features that you encounter: roads, fences, ditches, curbs, striping, etc.

4. Leave plenty of space on the notes for the unexpected. Cross sections can grow and you may need to add a section at a pipe crossing, ditch crossing, road intersection, etc.
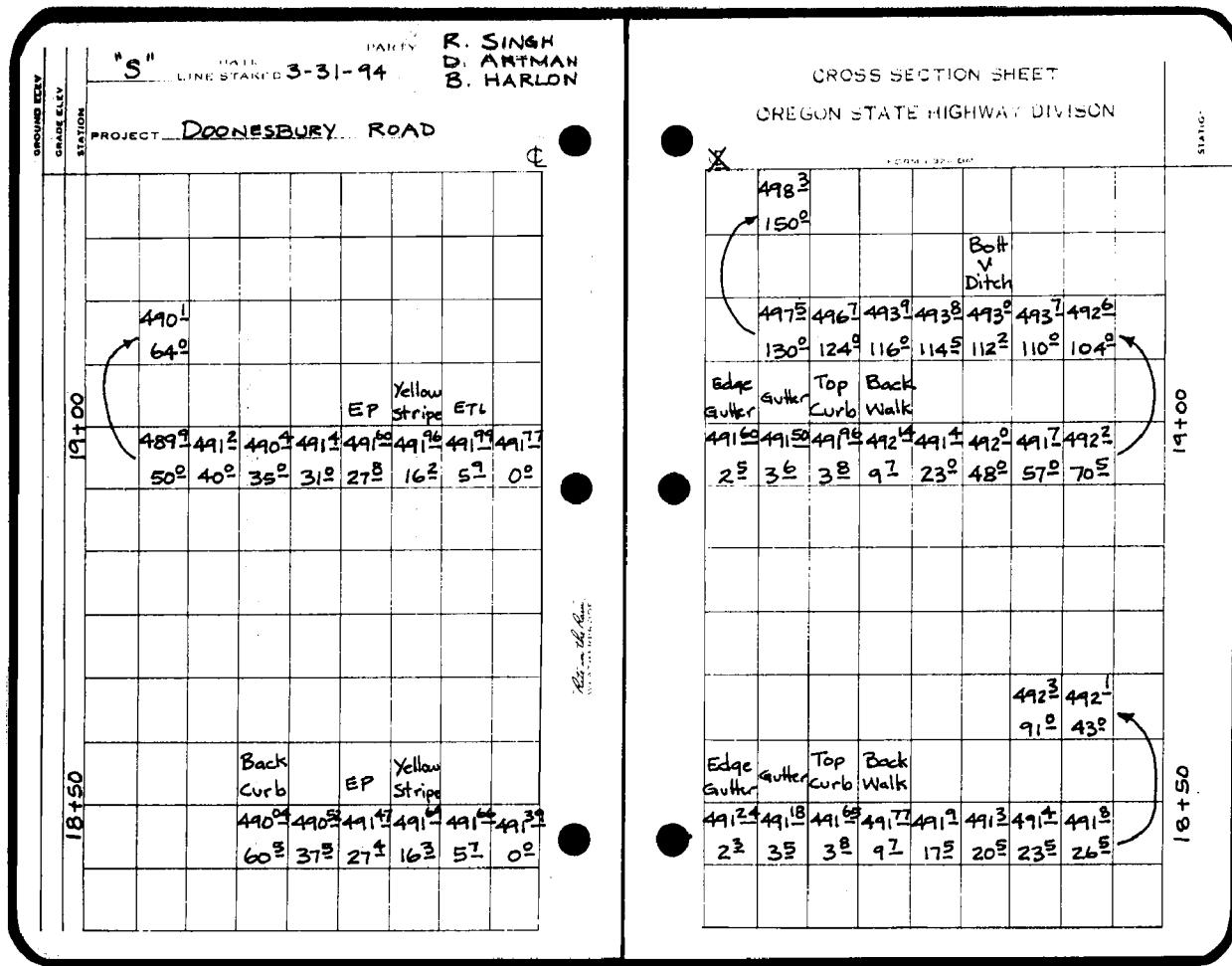
Cross sections can be taken from contour maps. The example shows a cross section picked from the contour map for station 1+50.
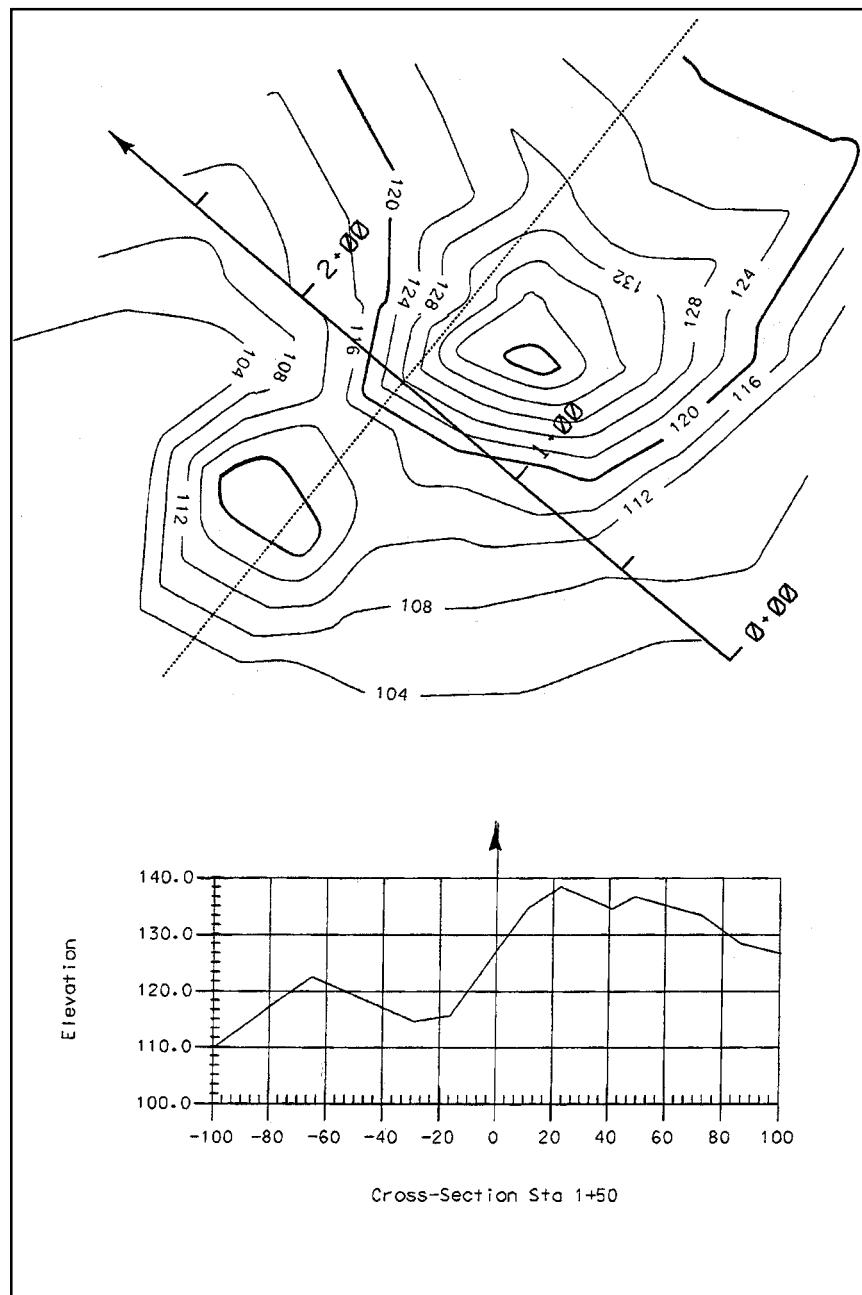
Figure 46

Digital Terrain Models

A digital Terrain Model (DTM) is numerical representation of the configuration of the terrain consisting of a very dense network of points of known X,Y,Z coordinates. Modern surveying and photogrammetric equipment enables rapid three dimensional data acquisition. A computer processes the data into a form from which it can interpolate a three dimensional position anywhere within the model.

Think of a DTM as an electronic lump of clay shaped into a model representing the terrain. If an alignment was draped on the model and a vertical cut made along the line, a side view of the cut line would yield the alignment's original ground profile. If vertical cuts were made at right angles to the alignment at certain prescribed intervals, the side views of the cuts would represent cross sections. If horizontal cuts were made at certain elevation intervals, the cut lines when viewed from above would represent contours.

A DTM forms the basis for modern highway location and design. It is used extensively to extract profiles and cross sections, analyze alternate design alignments, compute earthwork, etc.
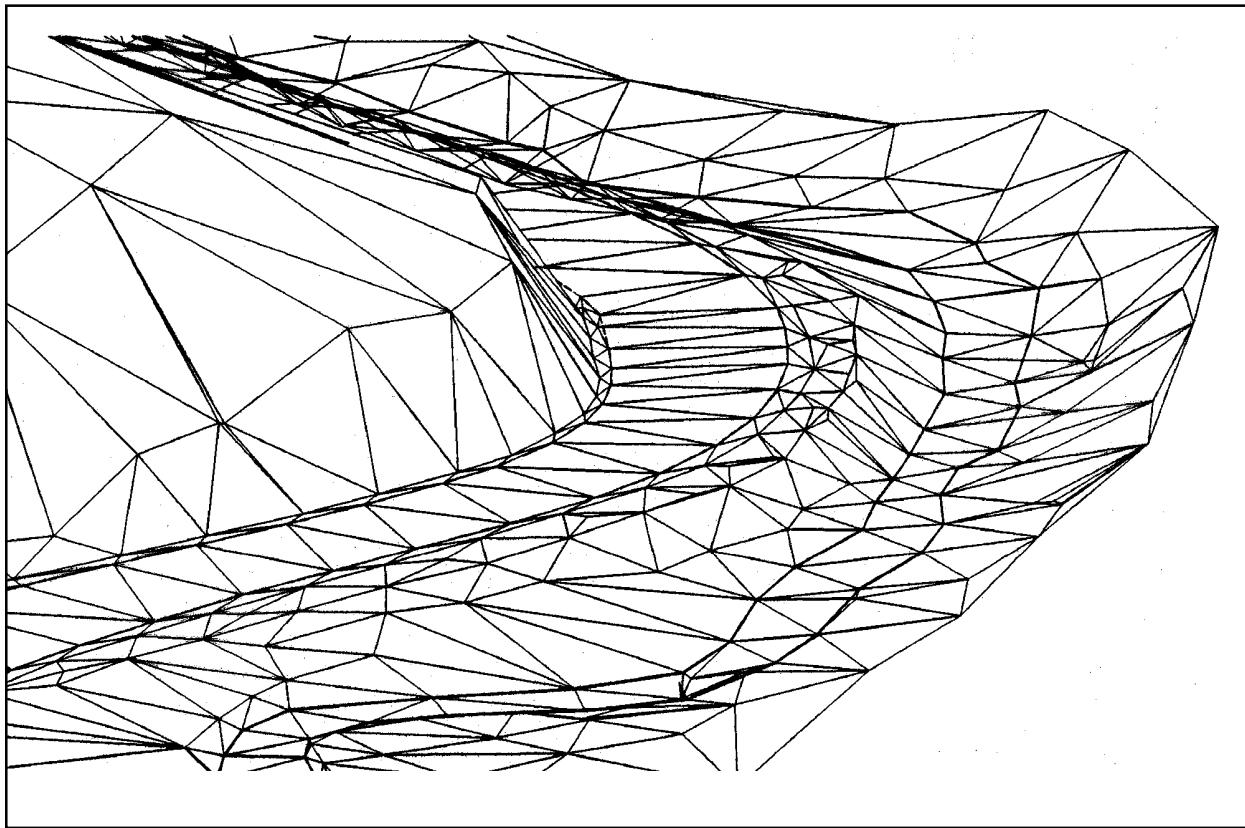
Figure 47

Example of Digital Terrain Model:

Horizontal Curves

Highway Curves

Many alignments are composed of one or more straight lines, or *tangent* alignments. Power lines, pipe lines, and low speed city streets are several examples. But for railroad or higher speed vehicular traffic, instantaneous changes in direction, either horizontally or vertically, are at best uncomfortable for the passengers and at worst, hazardous.

To lessen the forces involved when a vehicle changes direction, a gradual change is utilized resulting in a curve in the alignment. There are three general types of curves; Circular (or simple) curves, Spiral (or transition) curves, and Vertical (or parabolic) curves. This chapter will focus on the simple circular curve.

Stationing

One of the basic tasks of a survey crew is to layout or stake centerline and vertical alignments. One of the tools available to make this job easier is centerline stationing. Stationing is the assignment of a value representing the distance from some arbitrary starting point. Where the stationing begins is not generally too important, but any point along the alignment can be related to any other point on the same alignment by using the stationing.

A *station* is a linear distance of 100 feet along some described alignment. Without a described alignment, the station has no direction and therefore is rather meaningless.

Stationing is usually expressed as number of stations or 100 foot units plus the number of feet less than 100 and any decimal feet. This value is preceded by an alphanumeric alignment designation. A point on an alignment called B3 and 1345.29 feet from the beginning of the stationing would be designated as "B3 13+45.29". To perform math with stationing, the "+" can be dropped and the distance treated as feet.

Circular Curves

The simplest of the three curves is the circular or simple curve. The circular curve is exactly what the name implies, a segment of a circle. Circular curves are used for horizontal alignments because they can be laid out on the ground using basic surveying tools and techniques.

To layout a circular curve, the surveyor usually uses a chain or EDM to measure distances along the arc of the curve and a transit or theodolite to measure the horizontal angles from a reference line to the station to be set. Before laying out a circular curve we need to know it's parameters.

- The radius of the curve.
- The beginning station.
- The distances along the arc between the instrument and the points to be set.

Knowing these basics we can determine the deflection angles between stations.

From basic geometry we know that:

- Circumference = πD or 2πR

    Where, π = 3.1415926 and D = the Diameter of the circle, and R = the Radius.

- We also know that there are 360 degrees in a circle.

If we say that the length along the arc is L, then we can determine the interior angle of the circle subtended by the arc. To do that, let's calculate the fractional part of the arc to the circumference.

$$(Fractional\ Portion) = \frac{(Arc\ Length)}{(Circumference)} = \frac{L}{2\pi R}$$

This represents the fractional part of the total circle subtended by the arc. We will call the subtended angle Δ.

$$\frac{\Delta}{360°} = \frac{L}{2\pi R} \quad \therefore \quad \Delta = \frac{L \times 360°}{2\pi R}$$

We also know that the **deflection angle** α is 1/2 Δ, by combining terms we can write the deflection angle like so,

$$\alpha = \frac{360L}{4\pi R} = \frac{90L}{\pi R}$$

These are the formulas we would use to calculate the curve deflections for circular curves.

The *Degree of Curve* is defined as the angle subtended by an arc whose length is 100 ft.

A *Radian* is the angle subtended by an arc whose length equals the length of the **Radius**, or
57° 17' 44.8", or 57.295779513°.

Pi = π = 3.1415926
Circumference   = $2\pi R$
Degrees in a circle = 360°
Radius of a one degree curve = 5729.5779513 ft.
D = Degree of Curve.
R = Radius of the curve.
Δ = Delta, the central angle of the curve.
α = Alpha, the deflection angle to the point to be set.
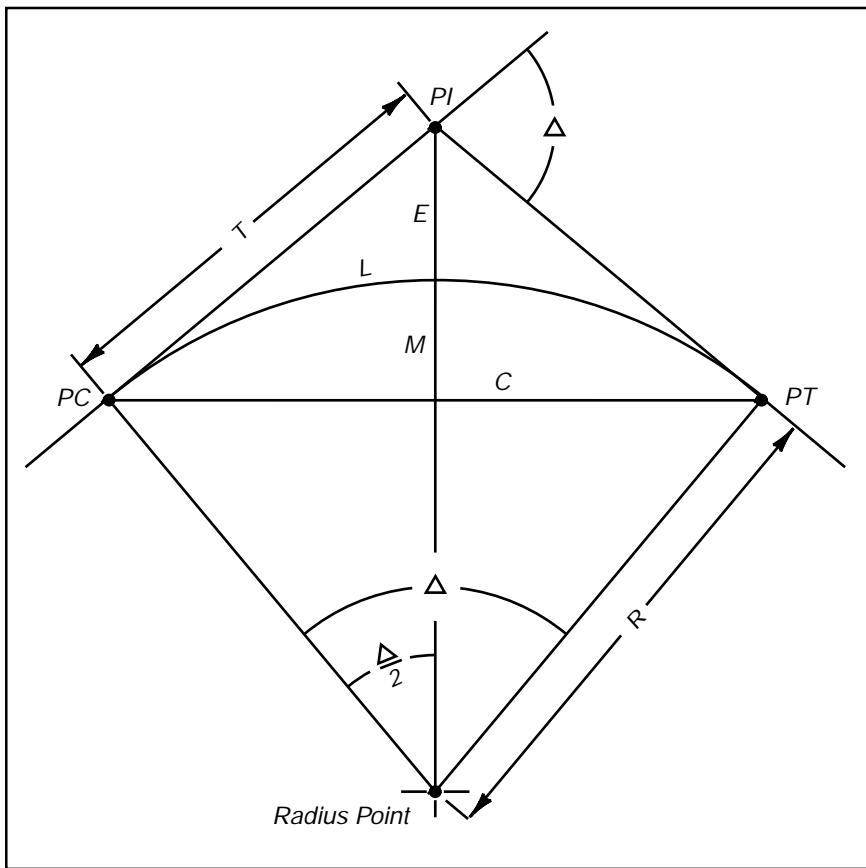
Figure 48

$$R = \frac{5729.578}{D} \qquad D = \frac{5729.578}{R}$$

$$T = R \cdot Tan \frac{\Delta}{2} \qquad C = 2R \cdot Sin \frac{\Delta}{2}$$

$$M = R(1\text{-}Cos \frac{\Delta}{2}) \qquad L = \frac{100\Delta}{D} = \frac{\pi R\Delta}{180°}$$

$$E = (\frac{R}{Cos \, \Delta/2})\text{-}R$$

Calculate the following horizontal curve elements:

Chord Length	
Degree of Curve	4° Lt
Delta	20°
External Distance	
Length of Curve	
Middle Ordinate	
P.C. Station	
P.I. Station	125+52.00
P.T. Station	
Radius	
Tangent Distance	

Calculate the curve layout data:

Station	Deflection	Long Chord

Calculate a 30' right offset curve for the same curve as that on page 13-5:

Chord Length	
Degree of Curve	
Delta	
Length of Curve	
P.C. Station	
P.I. Station	
P.T. Station	

Calculate the offset curve layout data:

Station	Deflection	Long Chord

Spiral Curves

Spiral Curve Definition

The Oregon Department of Transportation 1973 Standard Highway Spiral Manual gives the following definition for a spiral curve:

> *The Standard Highway Spiral is a curve whose degree varies directly as its length, beginning at zero at the P.S. and reaching a degree of curve equal to the simple curve at the P.S.C.*

In simple terms, a spiral is a curve whose radius keeps getting shorter, like a dog running around a tree with his chain getting shorter and shorter.

Spirals are sometimes called transition curves because they are used to transition into and out of circular curves.

The Purpose in using Spiral Curves

To understand the rational for using Spiral curves, we must take a brief look at the basic physics involved when a vehicle travels through a curve. The first figure below shows the forces at work as a truck negotiates a right-hand turn on a flat roadway. There is the force of gravity pulling the truck toward the center of the earth. There is also the centrifugal force caused by the continuous change in the direction required to successfully navigate a curve. This centrifugal force causes the truck to want to slide to the left off the roadway. If the friction of the tires on the roadway is sufficient to prevent this sideways slide, the centrifugal force then creates a torsional or rotating force that will try to tip the truck over. The sharper the curve, the greater this force will be at a given speed.
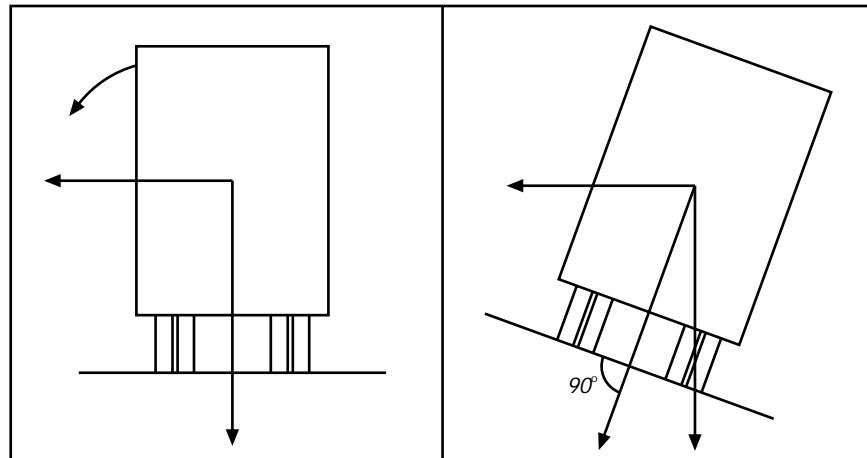
Figure 49

To lessen the effect of these potentially hazardous forces, roadways are super elevated or banked through corners. The super elevation is designed such that the road surface is near perpendicular to the resultant force of gravity and centrifugal inertia. The second figure shows this situation. However, in order to transition from a flat roadway to a fully super elevated section and still maintain the balance of forces, the degree or sharpness of the curve must begin at zero and increase steadily until maximum super elevation is reached. This is precisely what a Spiral Curve does.

Spiral Curve Nomenclature

The P.S. or "point of spiral" is the point of change from tangent to spiral. The P.S.C. or "point of spiral to curve" is the point where the radius of the spiral has decreased to match that of the circular curve. It is the point of change from spiral to circular curve. These terms apply to the spiral that is transitioning into the curve travelling ahead on line. On the outgoing spiral, the P.C.S. or "point of curve to spiral" and P.T. or "point of tangency" are mathematically identical to the P.S.C. and the P.S. respectively.

The difference in stationing between the P.S. and the P.S.C. is the length of the spiral. It is expressed as "L". The degree of curve of the circular curve and the length of the spiral dictates the rate of change in the radius of the spiral. This rate of change, known as "a", is the change in degree of curve per station (100 feet) of spiral or:

$$a = \frac{100D}{L}$$

Where: D = the degree of curve of the simple curve.
L = the total length of the spiral in feet

For a 5 degree curve with a 250 foot spiral

$$a = \frac{(100)\ (5)}{250} = 2$$

Exercise: Compute a for the following spirals.

	Degree of Curve	Length of Spiral	a value
1	10°	200'	
2	7° 30'	250'	
3	4°	500'	
4	1° 30'	350'	

$S°$ angle (or Delta "Δ") of a spiral curve.

Within a spiral curve some change in direction occurs reducing the Δ value of the central curve by some amount. Also this angle referred to as the $S°$ angle is needed in order to determine most of the other properties of the spiral.

In a simple curve, Δ is the degree of curve (**D**), times the length of curve (**L**) in stations, or:

$$\Delta = \frac{DL}{100}$$

Where: **D** = degree of curve
L = length of curve in feet

Since a spiral has a constantly variable **D**, beginning at zero and ending at **D** of the simple curve, $S°$ is the average degree of curve (or **$D/2$**) times the length of the spiral in stations, or:

$$S° = \frac{DL}{200}$$

Where: **D** = the degree of curve of the circular curve
L = the length of the spiral in feet

Exercise: Using this formula, determine the $S°$ for the following spirals:

	Given Information	s °
5	200 ′ spiral into a 4° curve	
6	400 ′ spiral into an **a** value of 1	
7	spiral with an **a** value of 2 into a 5° curve	
8	500 ′spiral into a 6° 15 ′ curve	

X and Y, the Ordinate and the Abscissa.

The ordinate, represented by **X**, and the abscissa, represented by **Y**, are the backbone of most of the calculations of spiral elements. The ordinate, **X**, is the right angle offset distance from the P.S.C. to a point on the tangent line. This point on the tangent is called the x-point. The abscissa, **Y**, is the distance along the tangent from the P.S. to the x-point.
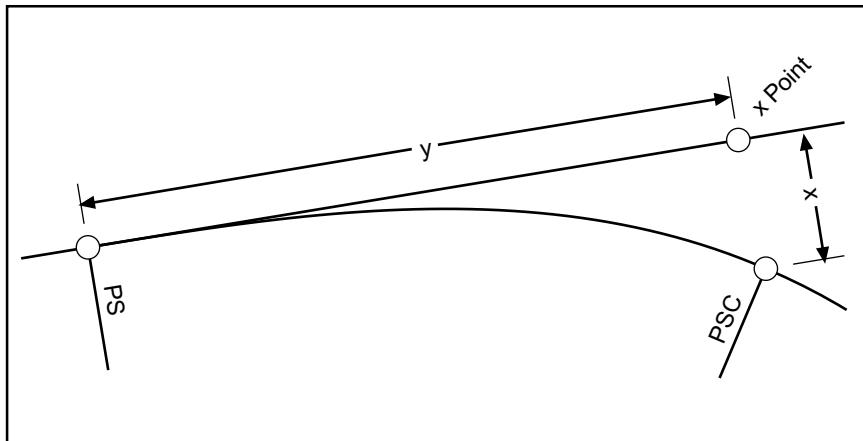
Figure 50

The calculation of **X** and **Y** is probably not something most of us would choose to do more than once without a computer. They are described as:

$X=LM$ and $Y=LN$

Where: **L** = Spiral Length
M & **N** each represent the summation of a different infinite series involving **S°** expressed in radians. These formulae can be found on pages 8 and 9 of the spiral manual.

Fortunately for us, somebody has calculated **M** and **N** for every minute of **S°** angle from 0° to 100° and recorded them in *Table IV* of the spiral manual.

Other Spiral Elements

Some elements that we need for designing, drafting, and field layout of spiral curves are listed on the next few pages.

The deflection from the tangent to the P.S.C.

$$i = a\tan(\frac{x}{y})$$

The chord from the P.S. to the P.S.C.

$$C = \sqrt{x^2 + y^2}$$
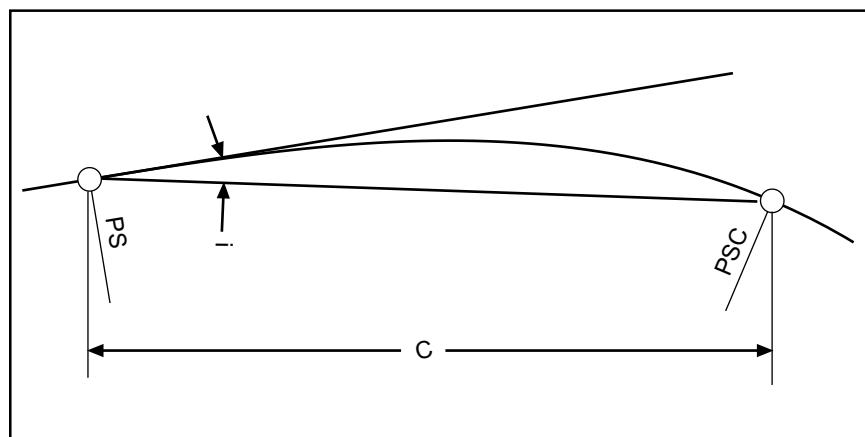
Figure 51

Besides **X** and **Y**, table IV also contains factors for **i**, **c**, **p**, **q**, **u** and **v**. All we have to do is multiply the values in Table IV by **L** to get to get these elements. All of the spiral elements are shown graphically in Figure 54.
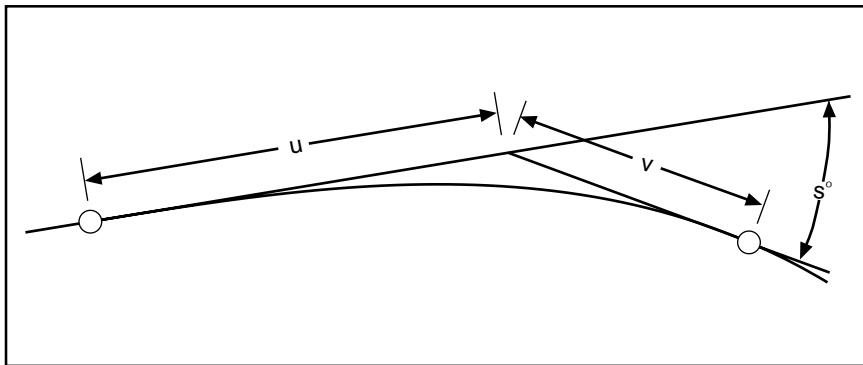
Figure 52

The long spiral tangent is represented by **u** and the short one by **v**, as shown above. These two lines intersect at an angle equal to **S°**.
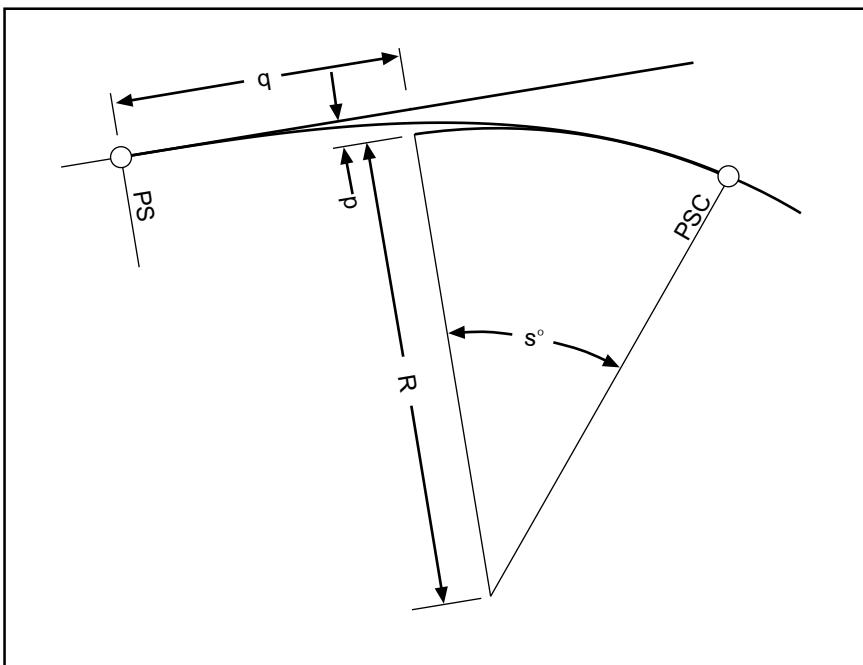
Figure 53

The offset from the main tangent to the point where the tangent to the circular curve becomes parallel to the main tangent is known as **p**. The distance along the tangent from the PS or PT to a point perpendicular from the radius point is called **q**.
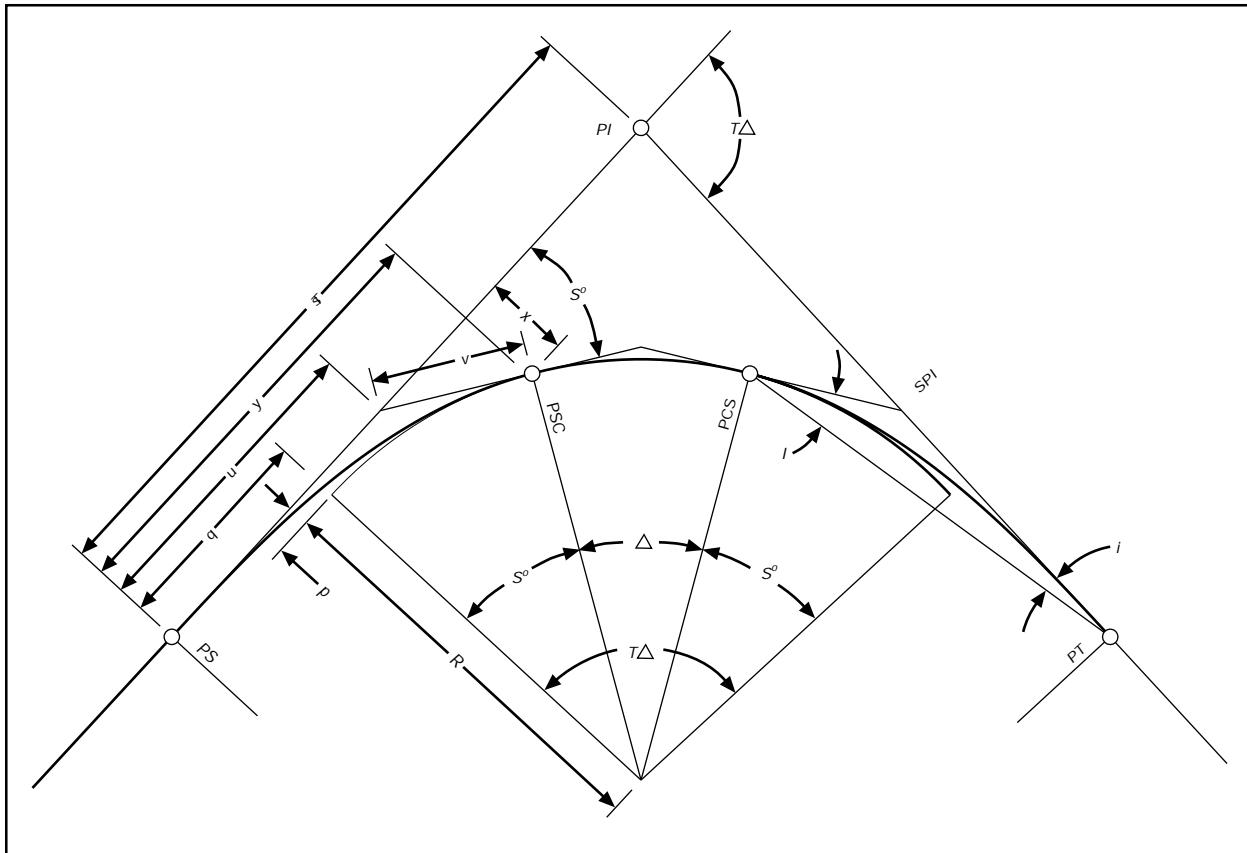
Figure 54

- The semi-tangent distance from the P.S. to the P.I. of the total curve, given equal spirals on each end.

$$SemiTangent = q + (R + p) \tan \tfrac{1}{2} \; \Delta$$

Where:

$R = radius$

$q = Y\text{-}R\sin S°$

$p = X\text{-}R(1 - \cos S°)$

Typical Solution of Spiral Elements

Given $L = 400$, $a = 0.5$, $T\Delta = 14°$

$$a = \frac{100D}{L}$$

Multiplying both sides by $L/100$ we get:

$$D = \frac{aL}{100} = \frac{(0.5)(400)}{100} = 2° \ 00'$$

And solving for the spiral angle and the radius of the circular curve:

$$S° = \frac{DL}{200} = \frac{(2)(400)}{200} = 4° \ 00'$$

$$R = \frac{5729.578}{2} = 2,864.79$$

Taking the following values from Table IV of the Spiral manual and multiplying both sides of the equation by L (400) we get:

i	=	1° 20'			
C/L	=	0.999783	C	=	399.91
Y/L	=	0.999513	Y	=	399.81
X/L	=	0.023263	X	=	9.31
P/L	=	0.005817	p	=	2.33
q/L	=	0.499919	q	=	199.97

and

$$SemiTangent = q + (R+p)\tan\frac{T\Delta}{2} = 199.97 + (2864.79 + 2.33)\tan\left(\frac{14°}{2}\right) = 552.01$$

Practice exercise:

9) Given $D = 6°$, $T\Delta = 45°$, and $L = 400'$ and using the excerpt from Table IV in the appendix, solve for a, $S°$, i, C, Y, X, and the semi-tangent.

Deflection Angles

It should be apparent that deflection angles for the spiral in exercise 9 could be computed by determining the **X** and **Y** values to each point to be staked or plotted and then use the formula:

$$i = atan \frac{X}{Y}$$

Another option is by calculating the **$S°$** to each point and then reading the deflection angle directly from Table IV of the Spiral manual. Table II also lists deflections in minutes on angle divided by **a**, for each foot of the spiral. We could read this table for each point and multiply by the **a** value of 1.5. A similar value can be interpolated from Table III.

However for most situations the following formula will give adequate results. Errors will be less than one minute if the **$S°$** is 25° or less. The greater the **$S°$** and the longer the spiral, the greater the error will be in the calculated position of the point.

$$\boldsymbol{i} = 10ad^2$$

Where:

   i = deflection in minutes.
   d = distance from the PS in stations.

or

$$\boldsymbol{i} = \frac{ad^2}{60,000}$$

Where:

   i = deflection in degrees
   d = distance from the PS in feet

Practice exercise:

10) Given a PS station at 321+11.50, compute deflections to even 50' stations on the spiral from the previous exercise. The deflection to station 321+50 is done for you.

   d = (Station 321+50) - (Station 321+11.50) = 38.50 feet

   i = $(1.5)(38.5)^2 / 60000$ = 0° 02' 13"

Vertical Curves

We have covered both the simple and spiral horizontal curves. These are used to give us stations along the alignment. Now we will look at the vertical alignment and the vertical curve. The vertical or parabolic curve gives us a smoother transition for elevations than either the simple or the spiral curve.

There is a simple method to calculate the elevation of any point along the vertical curve that uses the following procedure:

- Calculate the elevation along the tangent, at each station needed.
- Calculate the vertical offset for each station needed.
- Add the tangent elevation and the vertical offset for each station.
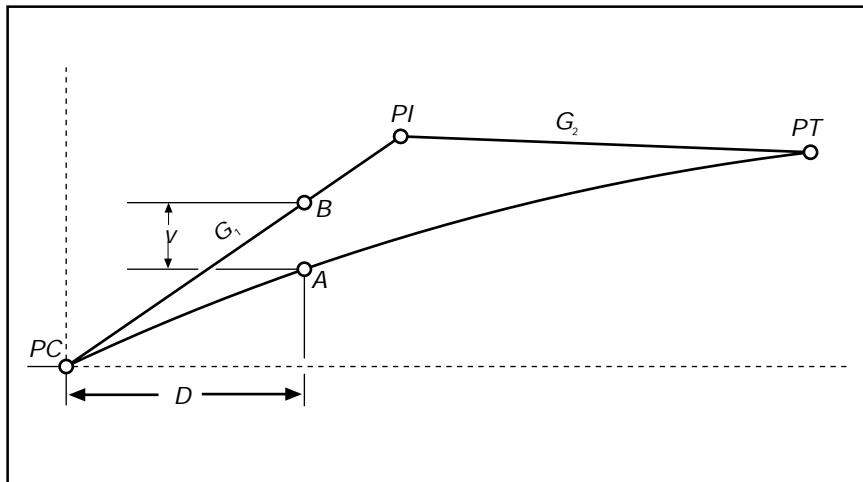
Figure 55

The Tangent Grade Elevation is calculated by using:

Tangent Elevation = (G1 * D) + PC Elevation

The Vertical Offset is calculated by:

$v = gD^2$

Where,

$$g = \frac{(G_2 - G_1)}{2L}$$

G_1=Grade of the back tangent
G_2=Grade of the ahead tangent
L=Length of the vertical curve
D=Distance from the PC to the station

Example of Crest Vertical Curve computation:
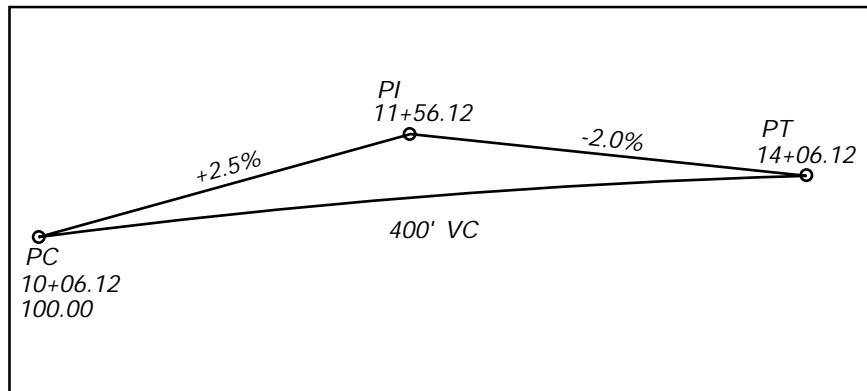
Figure 56

$$g = \frac{-0.020-0.025}{2(400)} = -0.00005625$$

Station	Distance	Tangent Elevation	V	Grade Elevation
10+06.12	0	100.00	0	100.00
10+50	43.88	101.10	-0.11	100.99
11+00	93.88	102.35	-0.50	101.85
11+50	143.88	103.60	-1.16	102.44
12+00	193.80	104.85	-2.11	102.74
12+50	243.88	106.10	-3.35	102.75
13+00	293.88	107.35	-4.86	102.49
13+50	343.88	108.60	-6.65	101.95
14+00	393.88	109.85	-8.73	101.12
14+06.12	400.00	110.00	-9.00	101.00

Exercise: Complete table below for Crest Vertical Curve
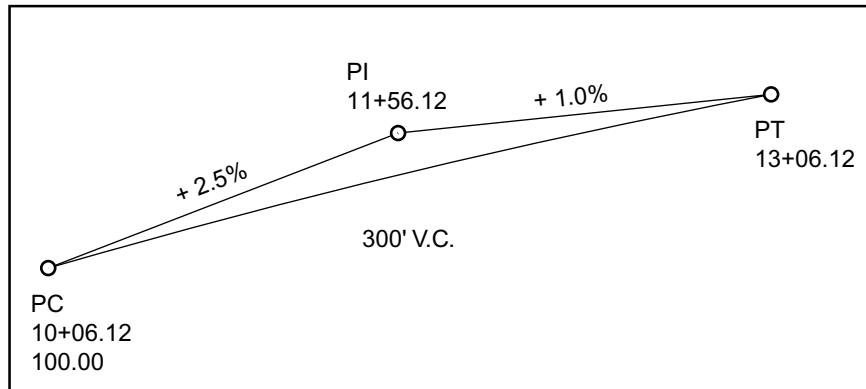
Figure 57

$$g = \frac{+0.01 - 0.025}{2(300)} = -0.000025$$

Station	Distance	Tangent Elevation	V	Grade Elevation
10+06.12				
10+50				
11+00				
11+50				
12+00				
12+50				
13+00				
13+06.12				

Exercise: Complete the table below for Sag Vertical Curve
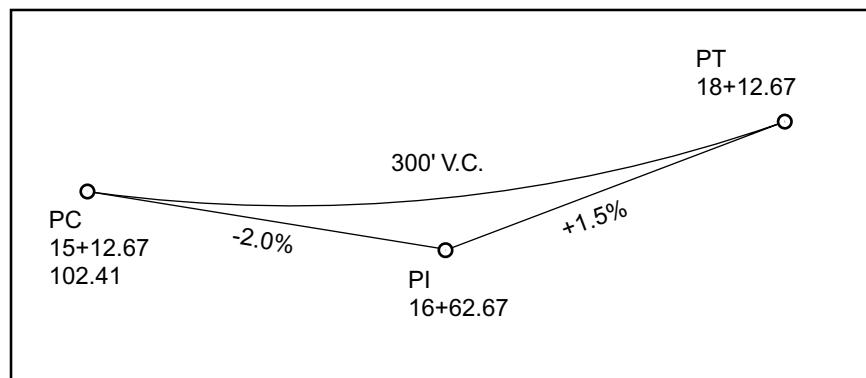
Figure 57

$$g= \frac{0.015+0.02}{2(300)} = 0.000058333$$

Station	Distance	Tangent Elevation	V	Grade Elevation
15+12.67				
15+50				
16+00				
16+50				
17+00				
17+50				
18+00				
18+12.67				

Law of Sines/Cosines

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\sin A = \frac{a*\sin B}{b} = \frac{a*\sin C}{c}$$

$$a = \frac{b*\sin A}{\sin B} = \frac{c*\sin A}{\sin C}$$
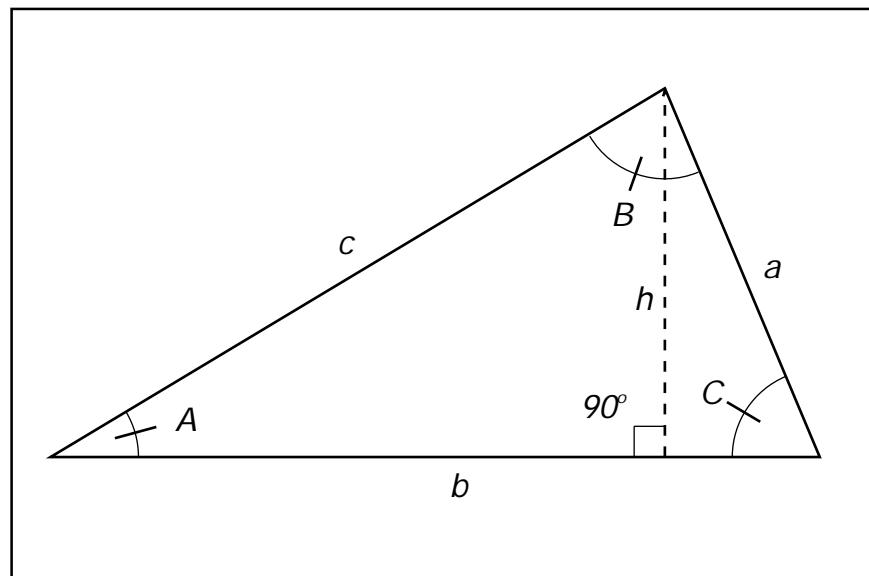
Figure 59

Law of Cosines

$$c^2 = a^2 + b^2 - 2ab* \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Derivation of the Pythagorean Theorem

Construct a rectangle of A width and B height. Next, create two right triangles by placing a diagonal, C. Note that the acute angles in each triangle must total 90 or it is not a right triangle.
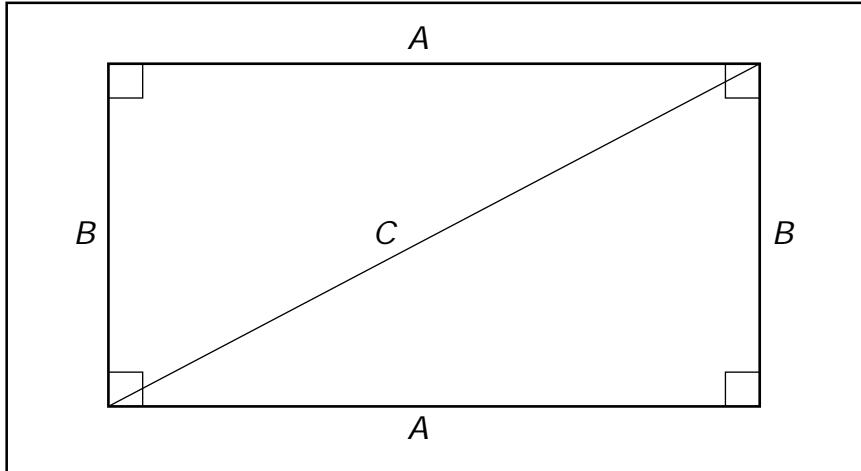
Figure 60

Now construct a square with sides C (equal to the hypotenuse of our right triangle). Place four copies of our triangle into the square as shown.
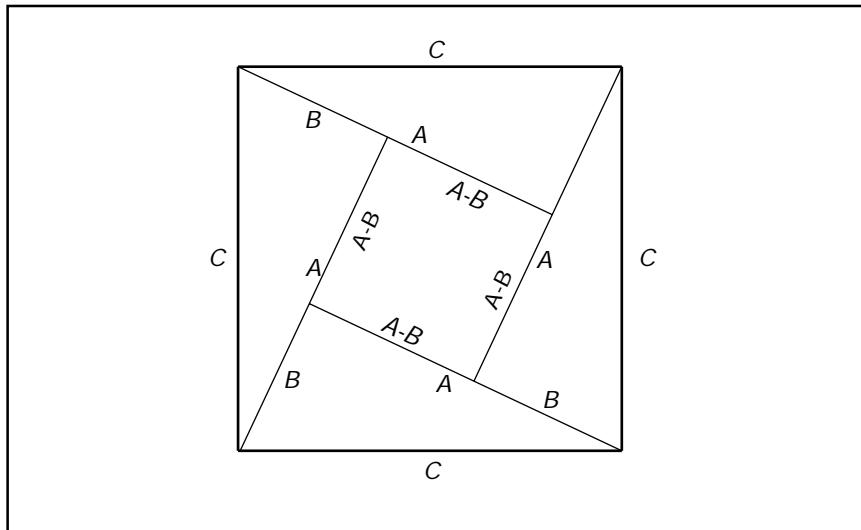
Figure 61

It can now be seen that the area of the larger square is equal to the area of the four triangles plus the area of the smaller square.

or $\quad C^2 = 4(\frac{1}{2} AB) + (A - B)^2$

$\quad C^2 = 2AB + (A^2 - AB - AB + B^2)$

$\quad C^2 = 2AB - 2AB + A^2 + B^2$

$\quad C^2 = A^2 + B^2$

Chaining Formulas

Slope Correction

$$H = s\cos\theta$$

where:
    H = Horizontal Distance
    s = Slope Distance
    θ = Vertical Angle

Temperature Correction

$$c_t = \alpha L(T - T_o)$$

where:
    α = Coefficient of thermal expansion (0.00000645 / 1° F)
    L = Measured Length
    T = Temperature of Chain
    T_o = Standard Temperature (68° F)

Tension Correction

$$C_p = \frac{(P-P_0)L}{aE}$$

where:
    C_p = Correction per distance L
    P = Applied Tension (Lb.s)
    P_o = Tension for which the tape was standardized.
    L = Length, (Ft.)
    a = Cross-Sectional Area of the Chain.
    E = Modulus of Elasticity of Steel. (30 * 10^6 lb/in²)

Sag Correction

$$C_s = \frac{w^2 L^2}{24P^2} \qquad \text{or} \qquad C_s = \frac{W^2 L}{24P^2}$$

where:
    C_s = Sag Correction between points of support, (Ft.)
    w = Weight of tape, (Lb.s / Ft.)
    W = Total Weight of tape between supports, (Lb.s)
    L = Distance between supports (Ft.)
    P = Applied Tension, (Lb.s)

Units of Measurement

1 Acre	=	43,560	Square Feet
	=	10	Square Chains
	=	4046.87	Square Meters
1 Chain (Gunter's)	=	66	Feet
	=	22	Yards
	=	4	Rods
1 Degree (angle)	=	0.0174533	Radians
	=	17.77778	Mils
	=	1.111111	Grads
1 Foot (U.S. Survey)	=	0.30480061	Meters *(1200/3937)*
1 Foot (International)	=	0.3048	Meters *(Exactly)*
1 Grad (angle)	=	0.9	Degrees
	=	0.01570797	Radians
1 Inch	=	25.4	Millimeters
1 Meter (m)	=	3.2808	Feet *(U.S. Survey)*
	=	39.37	Inches *(U.S. Survey)*
1 Mil (angle)	=	0.05625	Degrees
	=	3,037,500	Minutes
1 Mile (statute)	=	5280	Feet
	=	80	Chains
	=	320	Rods
	=	1.1508	Nautical Miles
	=	1.609347	Kilometers
1 Kilometer (km)	=	0.62137	Miles
1 Minute (angle)	=	0.29630	Mils
	=	0.000290888	Radians
1 Radian (angle)	=	57.2957795	Degrees
1 Rod	=	16.5	Feet
	=	1	Pole
1 Second (angle)	=	4.84814×10^{-6}	Radians
1 Square Mile	=	640	Acres
	=	27,878,400	Square Feet

Metric Prefixes

1 000 000 000 000 000 000 000 000	=	10^{24}	yotta Y
1 000 000 000 000 000 000 000	=	10^{21}	zetta Z
1 000 000 000 000 000 000	=	10^{18}	exa E
1 000 000 000 000 000	=	10^{15}	peta P
1 000 000 000 000	=	10^{12}	tera T
1 000 000 000	=	10^{9}	giga G
1 000 000	=	10^{6}	mega M
1 000	=	10^{3}	kilo k
100	=	10^{2}	hecto h
10	=	10^{1}	deka da
1	=	10^{0}	
0.1	=	10^{-1}	deci d
0.01	=	10^{-2}	centi c
0.001	=	10^{-3}	milli m
0.000 001	=	10^{-6}	micro μ
0.000 000 001	=	10^{-9}	nano n
0.000 000 000 001	=	10^{-12}	pico p
0.000 000 000 000 001	=	10^{-15}	femto f
0.000 000 000 000 000 001	=	10^{-18}	atto a
0.000 000 000 000 000 000 001	=	10^{-21}	zepto z
0.000 000 000 000 000 000 000 001	=	10^{-24}	yocta y

Commonly Used Constants

Coefficient of expansion of invar tape	=	0.0000001 per Degree Fahrenheit
Coefficient of expansion of steel tape	=	0.00000645 per Degree Fahrenheit
1 Degree of Latitude	=	69.1 Miles
1 Minute of Latitude	=	1.15 Miles
1 Second of Latitude	=	101 Feet
Length and Width of a Township	=	6 Miles or 480 Chains
Number of Sections in a Township	=	36
1 Normal Section	=	640 Acres
360 Degrees	=	400 Grads or 6400 Mils
Typical Stadia Ratio	=	100
Mean Radius of the Earth	=	20,906,000 Feet

Glossary of Terms

Benchmark: A fixed reference point or object, the elevation of which is known.

Contour: An imaginary line of constant elevation on the ground surface.

Deflection Angle: An angle between a line and the extension of the preceding line.

Departure: The departure of a line is its orthographic projection on the east-west axis of the survey. East departures are considered positive, West ones negative.

Height of Instrument: The height of the line of sight of the telescope above the survey station or control point (h.i). Sometimes referred to as the actual elevation of the telescope (H.I.).

Height of Target: The height of the target or prism above the survey station or control point. Sometimes the H.T. is referred to as the actual elevation of the target or prism.

Horizontal Angle: An angle formed by the intersection of two lines in a horizontal plane.

Horizontal Datum: The surface to which horizontal distances are referred and consists of (1) an initial point of origin, (2) the direction of a line from its origin, and (3) the polar and equatorial axes of the figure of the earth that best fits the area to be surveyed.

Horizontal Line: A line tangent to a level surface. In surveying, it is commonly understood that a horizontal line is straight, as opposed to a level line which follows the earths curved surface.

Hub: A Heavy stake (generally 2" x 2" x 12") set nearly flush with the ground with a tack in the top marking the exact survey point. An instrument is usually set up over this point.

Least Count: The smallest graduation shown on a vernier. This would allow the smallest possible measurement to be made on an instrument without interpolation.

Level Surface: A curved surface - every element of which is normal to a plumb line.

Latitude (traverse):   The latitude of a line is its orthographic projection on the north-south axis of the survey. North latitudes are considered positive, south ones negative.

Latitude (astronomical):   Angle measured along a meridian north (positive) and south (negative) from the equator, it varies from 0 degrees to 90 degrees.

Longitude: Angle measured at the pole, East or West from the Prime Meridian, varies from 0 degrees to 180 degrees East or 180 degrees West.

Meridian (astronomical):   An imaginary line on the earths surface having the same astronomical longitude at every point.

Meridian (magnetic):   The vertical plane in which a freely suspended magnetized needle, under no transient artificial magnetic disturbance, will come to rest.

Meridian (grid):   A line parallel to the central meridian or Y-axis of a rectangular coordinate system.

Pacing: A means of getting approximate distances by walking using steady paces. Under average conditions a person can pace with a relative precision of 1:200. Each two paces is called a stride. A stride is usually 5 feet, there would be roughly 1000 strides per mile.

Plumb Bob: A pointed metal weight used to project the horizontal location of a point from one elevation to another.

Range pole: Metal, wooden, or fiberglass poles used as temporary signals to indicate the location of points or direction of lines. Usually the pole is painted with alternate red and white one foot long bands. These poles come in sections and the bottom section shod with a steel point.

Stadia: A method of measurement to determine an approximate horizontal distance using the cross-hairs in a telescope and a leveling rod.

Vernier: A short auxiliary scale placed alongside the graduated scale of an instrument, by means of which the fractional parts of the least division of the main scale can be measured precisely.

Vertical Datum: The level surface to which all vertical distances are referred.

Vertical Line: A line perpendicular to the level plane.

Vertical Angle: An angle formed between two intersecting lines in a vertical plane. In surveying, it is commonly understood that one of these lines is horizontal.

Zenith Angle: An angle formed between two intersecting lines in a vertical plane where one of these lines is directed towards the zenith.

Glossary of Abbreviations

A	area
ac	acres
alt.	altitude
BM	bench mark
BS	back sight
BT	bearing tree
C	cut
CC	closing corner
Con. Mon.	concrete monument
const.	construction
decl.	declination
delta	central angle
dep.	departure
diam.	diameter
dir.	direct
dist.	distance
Dr.	drive
elev.	elevation
esmt.	easement
Ex.	existing
F	fill
F.H.	fire hydrant
F.L.	flow line (invert)
frac.	fraction

F.S.	foresight
G.M.	guide meridian
h.i.	height of inst. above sta.
HI	height of inst. above datum
H & T	hub and tack
hor.	horizontal
I.P.	iron pipe
I.R.	iron rod
L	left (x-section notes)
lat.	latitude
long.	longitude
max.	maximum
MC	meander corner
meas.	measurement
min.	minimum
M.H.	manhole
M.H.W.	mean high water
M.L.W.	mean low water
Mon.	monument
obs.	observer
obsn.	observation
orig.	original
pt.	point
pvmt.	pavement
R	right (x-section notes)
R, Rs	range, ranges

R 1 W	range 1 west
rev.	reverse
RP	reference point
R/W	right-of-way
SC	standard corner
S.G.	subgrade
spec.	specifications
Sq.	square
St.	street
sta.	station
Std.	standard
T, Tp, Tps	township, townships
TBM	temporary bench mark
temp.	temperature
T 2 N	township 2 north
TP	turning point
WC	witness corner
X-sect.	cross-section
yd.	yard

Example PPM Chart

Atmospheric correction (ppm)
at 60% relative humidity
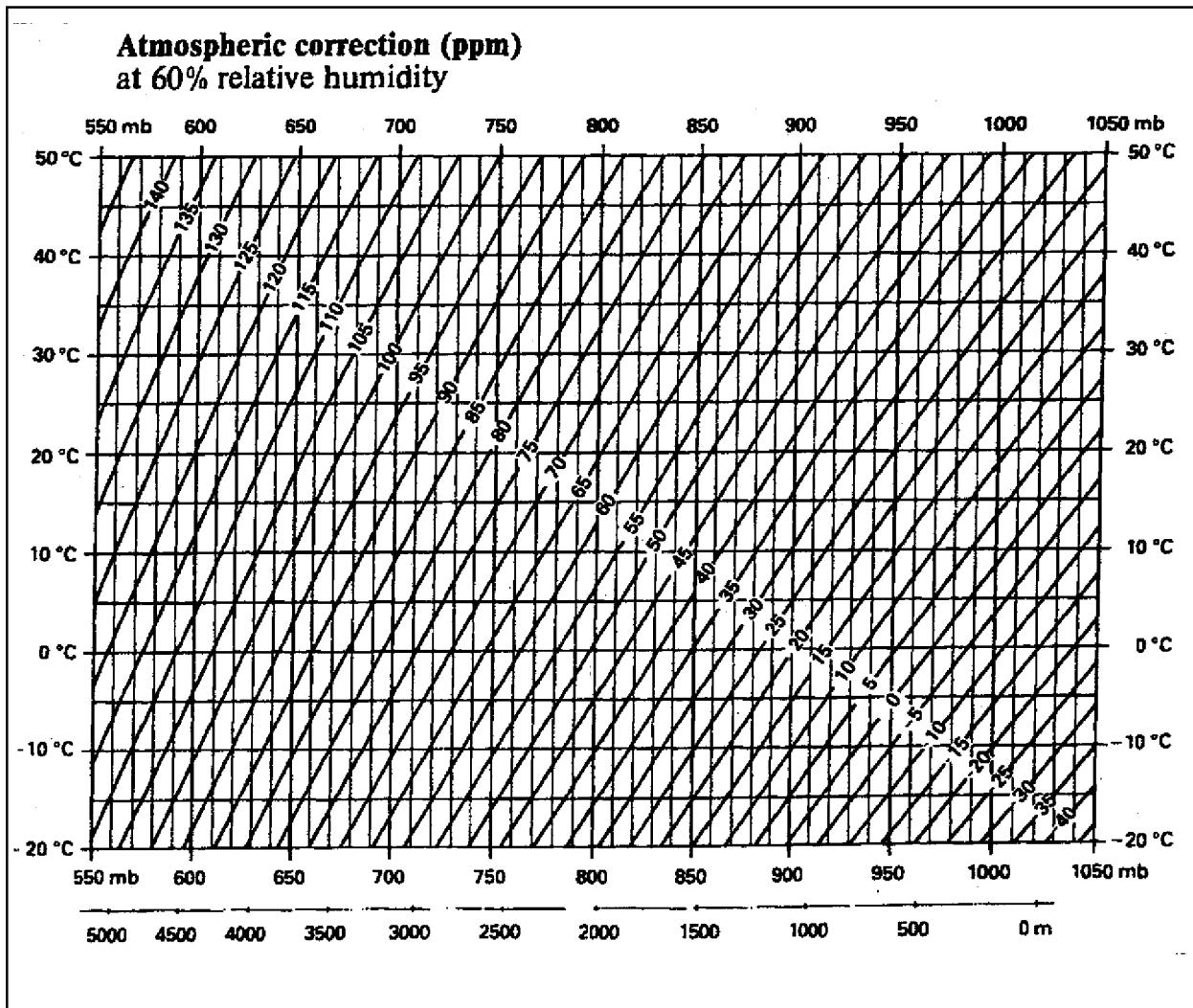
Example Traverse Calculation Sheet

Project:		Traverse ID:		Calculated By:		Date:
Purpose:		Page: of:		Checked By:		Date:

Point I.D.	Adjusted Azimuth	Horizontal Distance	Latitude	Departure	Adjustment Latitude	Adjustment Departure	Coordinates Northing	Coordinates Easting

Excerpt from Table IV (Spiral Book)

TABLE IV, FUNCTIONS OF THE STANDARD HIGHWAY SPIRAL

S	i	C/L	Y/L	X/L
11°31'	3°50'15"	0.998205	0.995967	0.066808
11°32'	3°50'35"	0.998200	0.995956	0.066904
11°33'	3°50'55"	0.998195	0.995944	0.067000
11°34'	3°51'15"	0.998190	0.995932	0.067096
11°35'	3°51'35"	0.998185	0.995921	0.067193
11°36'	3°51'55"	0.998179	0.995909	0.067289
11°37'	3°52'15"	0.998174	0.995897	0.067385
11°38'	3°52'35"	0.998169	0.995885	0.067481
11°39'	3°52'55"	0.998164	0.995874	0.067577
11°40'	3°53'15"	0.998158	0.995862	0.067673
11°41'	3°53'35"	0.998153	0.995850	0.067769
11°42'	3°53'55"	0.998148	0.995838	0.067865
11°43'	3°54'15"	0.998143	0.995826	0.067961
11°44'	3°54'35"	0.998137	0.995814	0.068057
11°45'	3°54'55"	0.998132	0.995803	0.068154
11°46'	3°55'15"	0.998127	0.995791	0.068250
11°47'	3°55'35"	0.998121	0.995779	0.068346
11°48'	3°55'55"	0.998116	0.995767	0.068442
11°49'	3°56'15"	0.998111	0.995755	0.068538
11°50'	3°56'35"	0.998106	0.995743	0.068634
11°51'	3°56'55"	0.998100	0.995731	0.068730
11°52'	3°57'15"	0.998095	0.995719	0.068826
11°53'	3°57'35"	0.998089	0.995707	0.068922
11°54'	3°57'55"	0.998084	0.995695	0.069018
11°55'	3°58'15"	0.998079	0.995683	0.069114
11°56'	3°58'35"	0.998073	0.995671	0.069210
11°57'	3°58'55"	0.998068	0.995659	0.069306
11°58'	3°59'15"	0.998063	0.995647	0.069402
11°59'	3°59'35"	0.998057	0.995635	0.069499
12°00'	3°59'55"	0.998052	0.995622	0.069595
12°01'	4°00'15"	0.998046	0.995610	0.069691
12°02'	4°00'35"	0.998041	0.995598	0.069787
12°03'	4°00'55"	0.998036	0.995586	0.069883
12°04'	4°01'15"	0.998030	0.995574	0.069979
12°05'	4°01'35"	0.998025	0.995562	0.070075
12°06'	4°01'55"	0.998019	0.995549	0.070171
12°07'	4°02'14"	0.998014	0.995537	0.070267
12°08'	4°02'34"	0.998008	0.995525	0.070363
12°09'	4°02'54"	0.998003	0.995513	0.070459
12°10'	4°03'14"	0.997997	0.995500	0.070555
12°11'	4°03'34"	0.997992	0.995488	0.070651
12°12'	4°03'54"	0.997986	0.995475	0.070747
12°13'	4°04'14"	0.997981	0.995463	0.070843
12°14'	4°04'34"	0.997975	0.995451	0.070939
12°15'	4°04'54"	0.997970	0.995438	0.071035
12°16'	4°05'14"	0.997964	0.995426	0.071131
12°17'	4°05'34"	0.997959	0.995414	0.071227
12°18'	4°05'54"	0.997953	0.995401	0.071323
12°19'	4°06'14"	0.997948	0.995389	0.071419
12°20'	4°06'34"	0.997942	0.995376	0.071515
12°21'	4°06'54"	0.997937	0.995364	0.071611
12°22'	4°07'14"	0.997931	0.995351	0.071707
12°23'	4°07'34"	0.997925	0.995339	0.071803

TABLE IV, FUNCTIONS OF THE STANDARD HIGHWAY SPIRAL

U/L	V/L	p/L	q/L	S
0.668083	0.334621	0.016726	0.499327	11°31'
0.668087	0.334625	0.016750	0.499325	11°32'
0.668091	0.334629	0.016774	0.499324	11°33'
0.668095	0.334633	0.016799	0.499322	11°34'
0.668100	0.334636	0.016823	0.499320	11°35'
0.668104	0.334640	0.016847	0.499318	11°36'
0.668108	0.334644	0.016871	0.499316	11°37'
0.668112	0.334648	0.016895	0.499314	11°38'
0.668116	0.334651	0.016919	0.499312	11°39'
0.668120	0.334655	0.016943	0.499310	11°40'
0.668124	0.334659	0.016967	0.499308	11°41'
0.668129	0.334663	0.016992	0.499306	11°42'
0.668133	0.334667	0.017016	0.499304	11°43'
0.668137	0.334670	0.017040	0.499302	11°44'
0.668141	0.334674	0.017064	0.499300	11°45'
0.668145	0.334678	0.017088	0.499298	11°46'
0.668150	0.334682	0.017112	0.499296	11°47'
0.668154	0.334686	0.017136	0.499294	11°48'
0.668158	0.334690	0.017161	0.499292	11°49'
0.668162	0.334693	0.017185	0.499290	11°50'
0.668167	0.334697	0.017209	0.499288	11°51'
0.668171	0.334701	0.017233	0.499286	11°52'
0.668175	0.334705	0.017257	0.499284	11°53'
0.668179	0.334709	0.017281	0.499282	11°54'
0.668184	0.334713	0.017305	0.499280	11°55'
0.668188	0.334717	0.017329	0.499278	11°56'
0.668192	0.334720	0.017354	0.499276	11°57'
0.668196	0.334724	0.017378	0.499274	11°58'
0.668201	0.334728	0.017402	0.499272	11°59'
0.668205	0.334732	0.017426	0.499270	12°00'
0.668209	0.334736	0.017450	0.499268	12°01'
0.668214	0.334740	0.017474	0.499266	12°02'
0.668218	0.334744	0.017498	0.499264	12°03'
0.668222	0.334748	0.017522	0.499262	12°04'
0.668226	0.334752	0.017547	0.499260	12°05'
0.668231	0.334756	0.017571	0.499258	12°06'
0.668235	0.334760	0.017595	0.499256	12°07'
0.668239	0.334764	0.017619	0.499254	12°08'
0.668244	0.334768	0.017643	0.499252	12°09'
0.668248	0.334771	0.017667	0.499249	12°10'
0.668252	0.334775	0.017691	0.499247	12°11'
0.668257	0.334779	0.017715	0.499245	12°12'
0.668261	0.334783	0.017740	0.499243	12°13'
0.668266	0.334787	0.017764	0.499241	12°14'
0.668270	0.334791	0.017788	0.499239	12°15'
0.668274	0.334795	0.017812	0.499237	12°16'
0.668279	0.334799	0.017836	0.499235	12°17'
0.668283	0.334803	0.017860	0.499233	12°18'
0.668288	0.334807	0.017884	0.499231	12°19'
0.668292	0.334811	0.017908	0.499229	12°20'
0.668296	0.334815	0.017933	0.499227	12°21'
0.668301	0.334819	0.017957	0.499225	12°22'
0.668305	0.334823	0.017981	0.499223	12°23'

Basic Trigonometry
Answer Key

1)	C	=	5				
2)	A	=	5				
3)	B	=	15				
4)	A	=	28				
5)	C	=	104				
6)	B	=	105				
7)	A	=	3.3				
8)	C	=	9.1				
9)	B	=	2.0				
10)	23.205°						
11)	42° 53' 06"						
12)	63° 32' 42"						
13)	87.980°						
14)	x	=	2				
15)	x	=	8				
16)	x	=	9				
17)	x	=	2.104				
18)	x	=	604.5				
19)	Sin A	=	3/5	=	0.6000		
	Cosine A	=	4/5	=	0.8000		
	Tangent A	=	3/4	=	0.7500		
	Cosecant A	=	5/3	=	1.6667		
	Secant A	=	5/4	=	1.2500		
	Cotangent A	=	4/3	=	1.3333		
20)	Sine A	=	42/70	=	3/5	=	0.6000
	Cosine A	=	56/70	=	4/5	=	0.8000
	Tangent A	=	42/56	=	3/4	=	0.7500
	Cosecant A	=	70/42	=	5/3	=	1.6667
	Secant A	=	70/56	=	5/4	=	1.2500
	Cotangent A	=	56/42	=	4/3	=	1.3333
21)	side c	=	13				
	Sine A	=	5/13	=	0.3846		
	Cosine A	=	12/13	=	0.9231		
	Tangent A	=	5/12	=	0.4167		
	Cosecant A	=	13/5	=	2.6000		
	Secant A	=	13/12	=	1.0833		
	Cotangent A	=	12/5	=	2.4000		
22)	cos A	=	0.9600				
	tan A	=	0.2917				
23)	sin 45°	=	0.7071				
	cos 45°	=	0.7071				
	tan 45°	=	1.0000				
24)	Same answers as #23.						
25)	sin 30°	=	0.5000				
	cos 30°	=	0.8660				
	tan 30°	=	0.5774				
26)	sinθ	=	3/5	=	0.6000		
27)	cosθ	=	4/5	=	0.8000		

28)	tanθ	=	3/4	=	0.7500
29)	sin 180° - θ	=	3/5	=	0.6000
30)	cos 180° - θ	=	-4/5	=	-0.8000
31)	tan 180° - θ	=	3/-4	=	-0.7500
32)	sin 180° + θ	=	-3/5	=	-0.6000
33)	cos 180° + θ	=	-4/5	=	-0.8000
34)	tan 180° + θ	=	-3/-4	=	0.7500
35)	sin 360° - θ	=	-3/5	=	-0.6000
36)	cos 360° - θ	=	4/5	=	0.8000
37)	tan 360° - θ	=	-3/4	=	-0.7500

38)

	Quad 1	Quad 2	Qaud 3	Quad 4
Sin	+	+	-	-
Cos	+	-	-	+
Tan	+	-	+	-

39)

	X=	Y=	Sine	Cosine	Tangent
0°	r	0	0/r = 0	r/r = 1	0/r = 0
90°	0	r	r/r = 1	0/r = 0	r/0 = ±∞
180°	-r	0	0/r = 0	-r/r = -1	0/r = 0
270°	0	-r	-r/r = -1	0/r = 0	r/0 = ±∞
360°	r	0	0/r = 0	r/r = 1	0/r = 0

40)	Adjacent side	=	5.1962
	Opposite side	=	3.0000
41)	36° 52' 12"+ and 53° 07'48"		
42)	Horizontal Distance = 85.99		
	Elevation Difference = +12.85		

Angle Measuring
Answer Key

Answers to exercises on page 5-11

STA	ANGLE RIGHT	
FS		
	1) 57° 02.2'	
	2) 114° 04.5'	
⊼	m) 57° 02.15'	
	6) 342° 13.7'	
	M) 57° 02.17'	
BS		
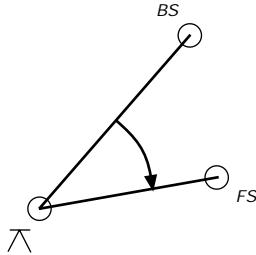
STA	PLATE READING	ADJ. PLATE READING	SET ANGLE	FINAL ANGLE
1 - (DIR) BS	0° 00' 00"	-00° 00' 03"		
4 - (REV) BS	179° 59' 54"		127° 57' 20.5"	
2 - (DIR) FS	127° 57' 14"	127° 57' 17.5"		
3 - (REV) FS	307° 57' 21"			
				127° 57' 16"
1 - (DIR) BS	359° 59' 58"	00° 00' 02.5"		
4 - (REV) BS	180° 00' 07"		127° 57' 11.0"	
2 - (DIR) FS	127° 57' 11"	127° 57' 13.5"		
3 - (REV) FS	307° 57' 16"			
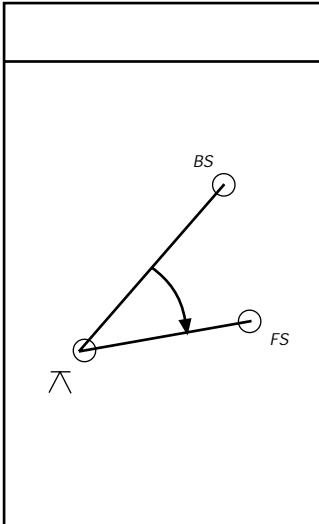
Direct Zenith Angle	102° 12 '45 "
Reverse Zenith Angle	257° 47 '21 "
Final Zenith Angle	102° 12 '42 "

Bearings and Azimuths
Answer Key

Answers to exercises for the following pages:

Page 6-7

Line	Azimuth
A – B	150°
B – C	103° 30′
C – D	349° 50′
D – E	50° 50′
E – F	164° 30′

Page 6-9

Line	Bearing
A – B	N 25° W
B – C	N 68° E
C – D	S 17° W
D – A	S 62° W

Page 6-12

Line	Bearing	Azimuth
A – B	S 55° E	125°
B – C	S 89° E	91°
C – D	N 49° E	49°
D – E	N 80° W	280°
E – A	S 64° W	244°

Coordinates
Answer Key

Answers to exercise on page 7-3:

Point	X	Y	Point	N	E
A	-5	6	D	-4	4
B	3	7	E	-8	0
C	2	1	F	-1	-6

Answers to exercise on page 7-7:

North

E

S 75° 57' 50" E
16.49

A

N 3° 00' 46" W
19.03

S 12° 59' 41" E
26.68

East

D

N 48° 00' 46" W
13.45

C

N 79° 41' 43" W
11.18

B

Perimeter - 86.83
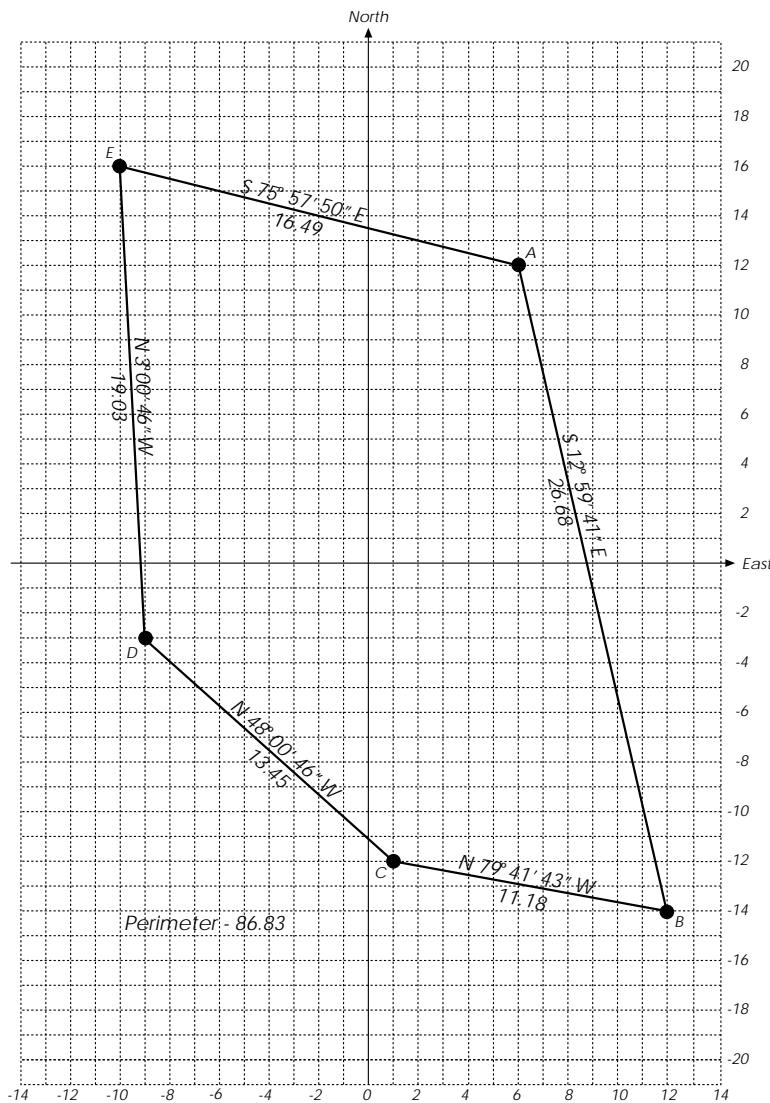
Answers to exercise on page 7-9:
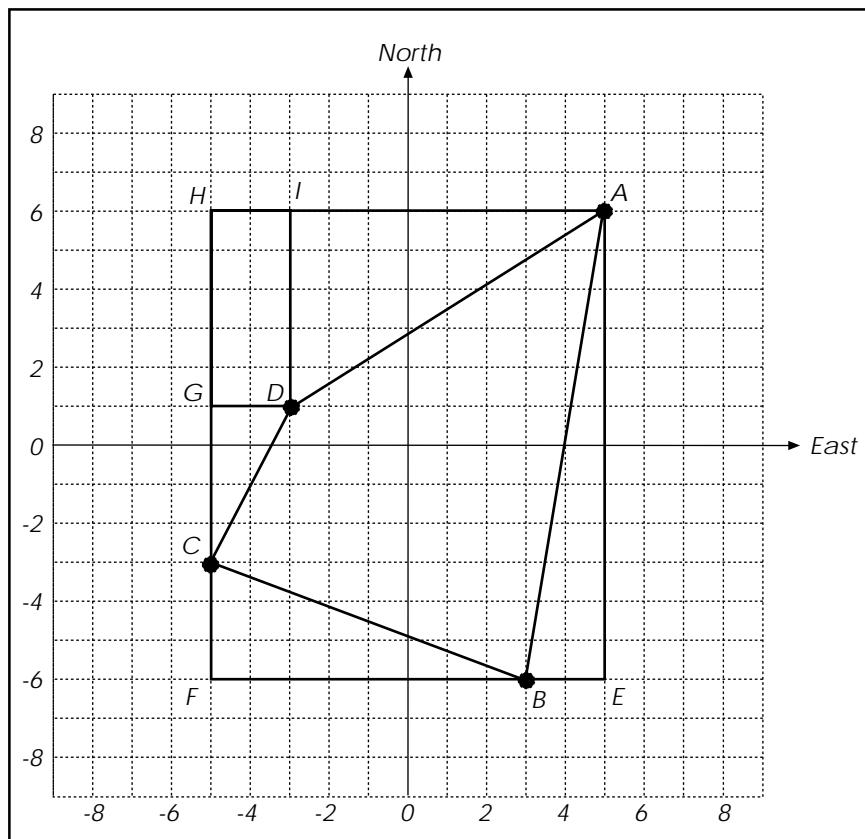
	Latitude	Departure
A-B	-12	-2
B-C	3	-8
C-D	4	2
D-A	5	8

Area	62 Square Units

A simple method to compute an area of a figure is to form a rectangle bounding the outermost points of the figure. Form rectangles, triangles, or trapezoids within the larger rectangle but outside the figure in question. The area of the figure is computed by subtracting the sum of the areas of the outer shapes from the area of the larger rectangle.

An alternate method is shown on the following page.

Alternate method to solve exercise on page 7-9:
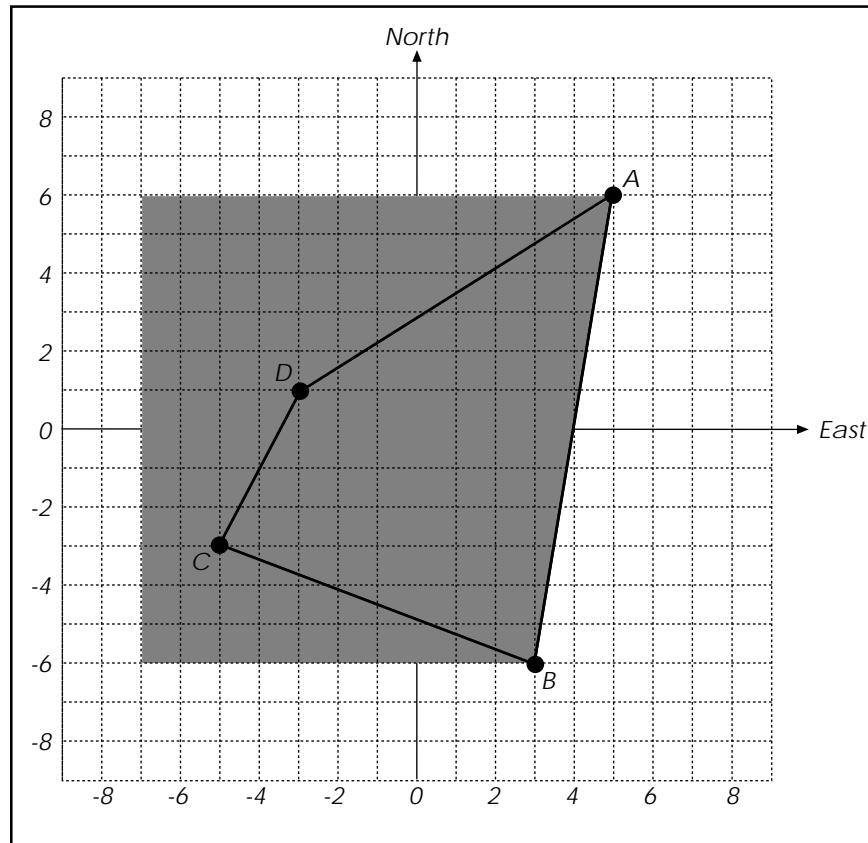
By establishing a baseline on the grid adjacent to the figure, several trapezoids can be formed between the baseline and points on the figure. To calculate the area within the figure subtract the areas of the trapezoids outside the figure from the area of the larger trapezoid formed by the baseline and the two furthermost points of the figure. See Area by Coordinates on page 7-8.

Answers to exercise on page 7-10:
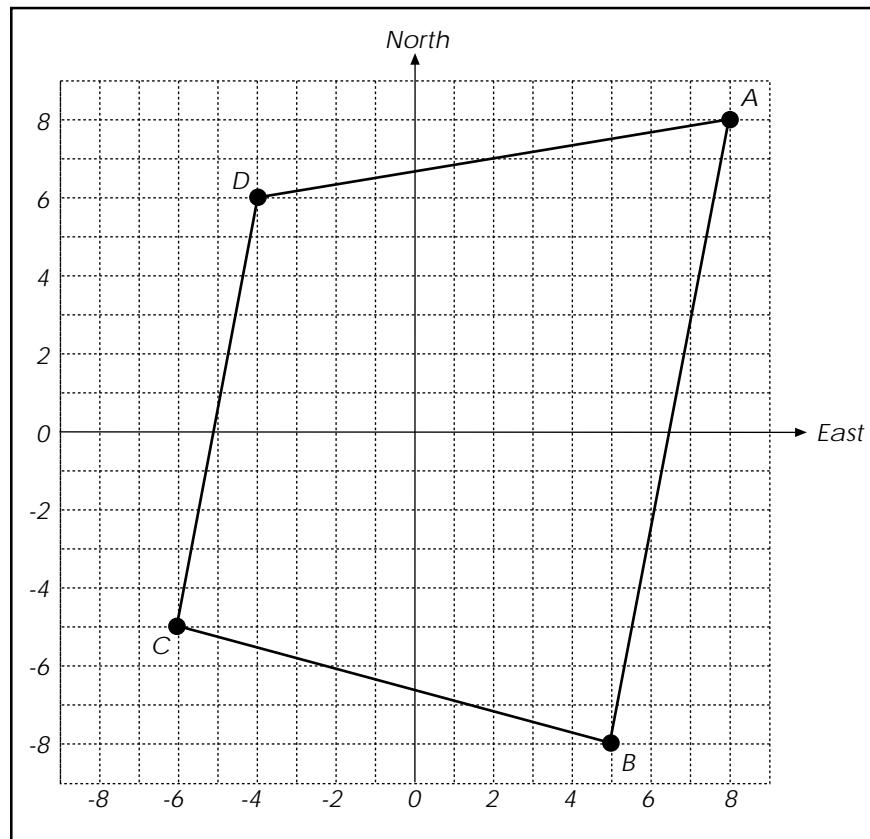
Point	Bearing	Distance	Latitude	Departure	Northing	Easting
A					8	8
	S 10° 37′ 11″ W	16.28	-16	-3		
B					-8	5
	N 74° 44′ 42″ W	11.40	3	-11		
C					-5	-6
	N 10° 18′ 17″ E	11.18	11	2		
D					6	-4
	N 80° 32′ 16″ E	12.17	2	12		
A					8	8

Traverse
Answer Key

Answers to exercise on page 8-9

Point ID	Measured angle	Measured Azimuth	Azimuth Adjustment	Adjusted Azimuth
X				
				270° 33' 57"
A	115° 19' 54"			
		25° 53' 51"	-0° 00' 03"	25° 53' 48"
B	227° 01' 24"			
		72° 55' 15"	-0° 00' 06"	72° 55' 09"
C	252° 48' 15"			
		145° 43' 30"	-0° 00' 09"	145° 43' 21"
D	304° 32' 06"			
		270° 15' 36"	-0° 00' 12"	270° 15' 24"
A	180° 18' 36"			
		270° 34' 12"	-0° 00' 15"	270° 33' 57"
X				

Answers to exercise on page 8-11

Point I.D.	Adjusted Azimuth	Horizontal Distance	Latitude	Departure	Adjustment		Coordinates	
					Latitude	Departure	Northing	Easting
	270°33'57"							
							3000.00	7000.00
	25°53'48"	1035.92	+931.896	+452.438	-0.022	-0.030		
							3931.874	7452.408
	72°55'09"	1415.50	+415.762	+1353.064	-0.031	-0.040		
							4347.605	8805.432
	145°43'21"	1645.55	-1359.750	926.776	-0.035	-0.047		
							2987.820	9732.161
	270°15'24"	2732.11	+12.239	-2732.083	-0.059	-0.078		
							3000.00	7000.00
	270°33'57"							
	Totals	6829.08	0	0	+0.147	+0.195		

Differential Leveling
Answer Key

B-13

Answers to exercise on page 10-4

Sta	BS (+)	H.I.	FS (-)	Elev	Description
BM X	6.72			935.42	BM X: Brass Disk in walk
		942.14			N.W. corner 12th and Sunset
TP 1	7.13		2.18	939.96	Elev: 935.42
		947.09			
TP 2	4.19		3.23	943.86	
		948.05			
TP 3	6.72		5.11	942.94	
		949.66			
TP 4	1.09		7.23	942.43	
		943.53			
BM Y			2.36	941.16	BM Y: 1/2" Iron Rod
					S.E. Corner 18th and Sunrise
					Elev: 941.19

Σ Backsights = 25.85
Σ Foresights = 20.11
Difference = 5.74

Check:

Begin Elev. = 935.42
BS/FS Diff. = +5.74
Ending Elev. = 941.16

True Elev = 941.19
Closing Elev. = 941.16
Misclosure = 0.03

Geometronics • February 2000   B-13

Trigonometric Leveling
Answer Key

Answers to exercise on page 11-4

Elevation of Point A	506.78 ft
Height of Instrument	5.21 ft
Height of Target	5.46 ft
Measured Slope Distance	837.58 ft
Direct Zenith Angle	78°37'42"
Reverse Zenith Angle	281°22'28"

Zenith Angle Reduction

Direct Zenith Angle	78°37'42"
Reverse Zenith Angle	281°22'28"
Sum	360°00'10"
360° Minus Sum	-0°00'10"
Half Value (error)	-0°00'05"
Plus Original Angle	78°37'42"
Final Zenith Angle	-78°37'37"

Curature and Refraction

Sight Distance in Miles (837.58/5280)	0.1586
Curvature and Refraction $(0.574)(0.1586)^2$	0.01 ft

Solving for Elevation at D

Elevation at B (tilting axis)	Elevation at A plus H.I.	511.99
Elevation difference between B & C	Slope Distance x COS (zenith angle)	165.17
Elevation at C (target tilting axis)	Elevation at B plus elevation difference	677.16
Elevation at D (without C&R)	Elevation at C minus height of target	671.70
Elevation at D (adjusted for C&R)	Elevation at D minus C&R	671.69

Horizontal Curves
Answer Key

Answers to exercise on page 13-5

Chord Length	497.47
Degree of Curve	4° Lt
Delta	20°
External Distance	22.10
Length of Curve	500
Middle Ordinate	21.76
P.C. Station	122+99.43
P.I. Station	125+52.00
P.T. Station	127+99.43
Radius	1432.40
Tangent Distance	252.57

Station	Deflection	Long Chord
127+99.43 P.T.	10°00'00"	497.47
127+50	9°00'41"	448.71
127+00	8°00'41"	399.27
126+50	7°00'41"	349.70
126+00	6°00'41"	300.02
125+50	5°00'41"	250.25
125+00	4°00'41"	200.41
124+50	3°00'41"	150.50
124+00	2°00'41"	100.55
123+50	1°00'41"	50.57
123+00	0°00'41"	0.57
122+99.43 P.C.	0°00'00"	0.00

Answers to exercise on page 13-6

Chord Length	507.88
Degree of Curve	3°55'05"
Delta	20°
Length of Curve	510.47
P.C. Station	122+99.43
P.I. Station	127+99.43
P.T. Station	1462.40

Station	Deflection	Long Chord
127+99.43 P.T.	10°00'00"	507.88
127+50	9°00'41"	458.11
127+00	8°00'41"	407.63
126+50	7°00'41"	357.02
126+00	6°00'41"	306.30
125+50	5°00'41"	255.49
125+00	4°00'41"	204.60
124+50	3°00'41"	153.65
124+00	2°00'41"	102.66
123+50	1°00'41"	51.63
123+00	0°00'41"	0.58
122+99.43 P.C.	0°00'00"	0.00

Spiral Curves
Answer Key

1) a = 5

2) a = 3

3) a = 0.8

4) a = 0.4

5) S = 4° 00'

6) S = 8° 00'

7) S = 6° 15'

8) S = 15° 37' 30"

9) a = 1.5
    S = 12° 00'
    i = 3° 59' 55"
    C = 399.22'
    Y = 398.25'
    X = 27.84'
    T = 598.14'

10) 321+50   i = 0° 02' 13"
    322+00   i = 0° 11' 45"
    322+50   i = 0° 28' 46"
    323+00   i = 0° 53' 18"
    323+50   i = 1° 25' 19"
    324+00   i = 2° 04' 51"
    324+50   i = 2° 51' 52"
    325+00   i = 3° 46' 24"
    325+11.50   i = 4° 00' 00"

Vertical Curves
Answer Key

Answers to exercise on page 15-3:
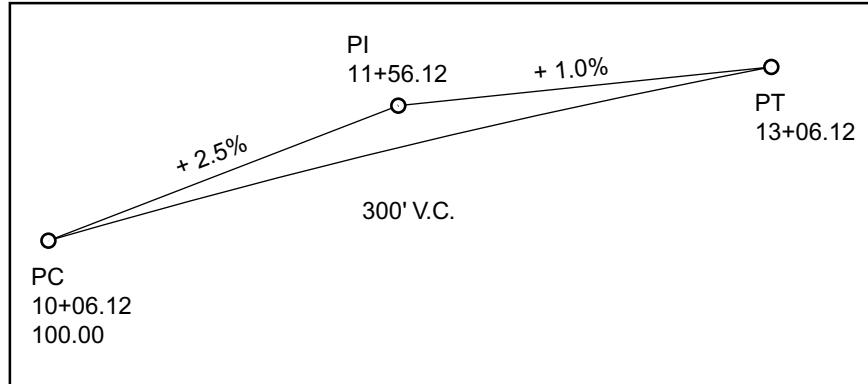
```
                    PI
                    11+56.12          + 1.0%
                       o                        o
                                                PT
          + 2.5%                                13+06.12

                            300' V.C.
      o
      PC
      10+06.12
      100.00
```

$$g= \frac{+0.01-0.025}{2(300)} = -0.000025$$

Station	Distance	Tangent Elevation	V	Grade Elevation
10+06.12	0	100.00	0	100.00
10+50	43.88	101.10	-0.05	101.05
11+00	93.88	102.35	-0.22	102.13
11+50	143.88	103.60	-0.52	103.08
12+00	193.88	104.85	-0.94	103.91
12+50	243.88	106.10	-1.49	104.61
13+00	293.88	107.35	-2.16	105.19
13+06.12	300.00	107.50	-2.25	105.25

Answers to exercise on page 105:
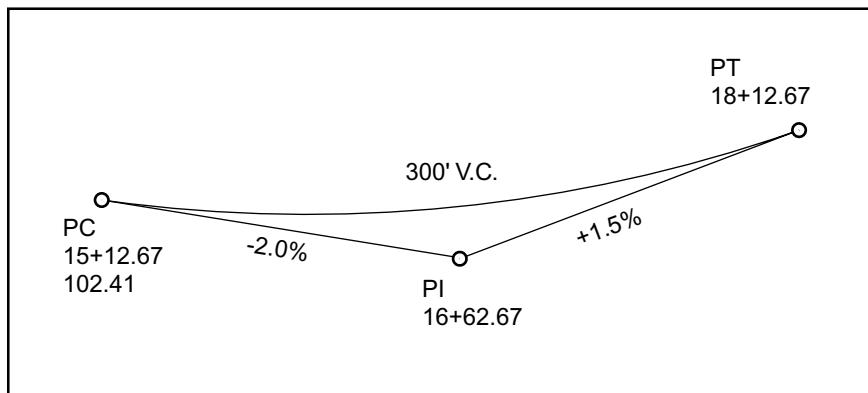
$$g = \frac{0.015 - 0.02}{2(300)} = 0.000058333$$

Station	Distance	Tangent Elevation	V	Grade Elevation
15+12.67	o	102.41	0	102.41
15+50	37.33	101.66	0.08	101.74
16+00	87.33	100.66	0.44	101.10
16+50	137.33	99.66	1.10	100.76
17+00	187.33	98.66	2.05	100.71
17+50	237.33	97.66	3.29	100.95
18+00	287.33	96.66	4.82	101.48
18+12.67	300.00	96.41	5.25	101.66

Surveyor's Conference Notes

Acknowledgement